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Linguistic Conceptualizations of Number

Full Appendix: Complete Transcriptions

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Complete Transcriptions

Data collection: International School Braunschweig (May/June, August 2013)

Class levels: Grade 2 – Grade 9

Bold lines in between the data sets within one class level are placed to separate different days of data collection.

Glossary of transcription symbols:

T	Teachers' utterances
T2	Second teacher's utterances
S (1, 2, 3 …)	Students' utterances
Stds	multiple students' utterances
(*abc*)	Not clearly intelligible utterance
(?)	Unintelligible utterances
(…)	Irrelevant parts left out
(.)	Pause
-	Trailing off
.	Declaration
!	Exclamation/Imperative
?	Question
(xx:yy)	Time of the utterance, relative to the start of audio recording

Class level: Grade 2

June 2013

T (02:15)	How are you gonna check? What are you gonna ask?
S1	Like, how many eights are in sixteen.
T	that was division. How about multiplication? How do check times?
S2	like eight times two is- ?

S3 (09:00)	how many fours are in thirty-two?
S4	eight
S4 (09:40)	eight times four
S3	(.) ehm, twenty-eight.
S4	four times eight
S3	like four times eight (?) is thirty-two

T (03:55)	How many sevens are there in sixty-three?
S1	sixty-three? (.) seven.
T	how many sevens are there in sixty-three?
S1	ah, nine.
T	mmh, if you say seven, you can always check, right? How can you check if there are seven sevens in sixty-three? How can you check?
S1	I can look, like, ten times seven is seventy and then I just take seven away and (then I know)
T	mmh, it can't be seven sevens, right, because seven sevens are forty-nine. right, ok

T (05:40)	four times nine
S2	(.) forty-five

T	four times nine
S2	thirty-six.
T	mmh, how do you know that four times nine is thirty-six? What helps you to solve the problem?
S2	four time- I know that nine times three, it is twenty-seven and plus another nine is thirty-six.
T (06:17)	seven times nine
S2	seven times nine? Six times nine is, eh, six times nine is fifty-four and plus another nine- (.) ok, what did you ask again?
T	seven times nine.
S2	seven times nine-
T	you said six times nine is fifty-four.
S2	then it's fifty-four plus another nine. fifty-four plus four- we can do
T	mmh, fifty-four plus nine.
S2	we can do that is not so tricky, we can do plus four, fifty-four plus four is, is fifty-eight and fifty-eight plus five- fifty-eight plus five is (.) thirty-six.
T (08:40)	What helps you to solve the problem seven times nine? What do you think?
S2	seven times nine. Six times- ehm, six times nine is (.)fifty-four. and fifty-four plus another nine is, is, thirty-six

T (19:00)	seven times seven
S3	seven times seven (.) wait (.) (five) plus seven is- forty-two, forty-nine!
T	eight times eight
S3	eight times eight. Ehm, nine times eight is, nine times eight is seventy-two minus eight is (.) sixty-four.
T (20:25)	six times seven
S3	six times seven. Wait, six times seven is- (…) six times five is- forty- ? (eight) six times seven is forty three.

T how do you know that six times seven- Why do you think that six times seven is forty-three?

S3 six times seven? Six times seven- ehm because seven- six times (seven times) or six times seven?

T seven times six or six times seven? Does it make a difference?

S3 because, ehm,

T does it make a difference?

S3 because six times five is fifty

T six times five is fifty?

S3 eh, thirty! Then thirty-six and thirty-six plus six equals forty- equals twelve. And then twelve plus thirty equals forty-two.

T aha, so six times seven equals?

S3 forty-two.

T (27: 20) How many eights are there in sixty-four?

S3 How many eights in sixty-four!? Wait! sixty-four, in sixty-four. how many sixes?

T how many eights.

S3 eights in sixty-four? (four, eight, ?) eight!

T how do you know that?

S3 because eight times nine is seventy-two.

T mmh! That's what you know already! Right? Ok!

S3 because nine times is so easy for me! And then just minus eight, so, ehm, seventy-two minus two is seventy and then we have (still) six left, so seventy minus six is sixty-four.

T (29:30) two times eight

S4 sixteen

T three times eight

S4	eh (.) don't know
T	ok, you just had two times eight is sixteen, right? And three times eight?
S4	twenty-four!
T	how do you know it's twenty-four?
S4	Because I done, it's, (I done a four) and then I added another four and that makes twenty-four. So I've done another eight.
T (37:00)	(.) seven times nine. What might help you to solve the problem seven times nine? What might help you? Mmh? Seven times nine?
S5	(.) I take ten, ehm from ninety, ehm nine away
T	from ninety? Seven times-
S5	ya
T	nine is close, close to-
S5	ten
T	ten. So if you're not too sure what's seven times nine, maybe you can ask yourself, oh, I know what's seven times ten. What is seven times ten?
S5	seven times ten equals seventy
T	and seven times nine is one (.) seven (.)
S5	is one seven less
T	less, so seventy minus seven?

Class level: Grade 3

June 2013

T (02:20)	What number must I add to sixty-nine to get hundred?
S1	thrity-one?
T	ok, can you explain how you found it out?
S1	ehm, I did hundred minus sixty-nine.
T	ok, is there another way? (.) and then you did what? Hundred minus sixty-nine!? Did you do it step by step or?
S1 (03:00)	yeah, I did step by step, hundred minus sixty-nine, I did first hundred minus sixty is forty, and then I did first forty minus ten is thirty plus, plus one, because its just nine, then it's thirty-one
T	ok, who has another way?
S2 (03:15)	I did it, like, I added up to the next tens which is seventy, I needed to only add one. Then, seventy, I, I just like added up, up, to hundred, and that was thirty, so thirty-one!
(…)	
S3 (05:00)	you always like do up to the next ten, like, with sixty-nine, there you do up to the next ten is one, and then you do it like, plus ten and so on and so on

T (08:00)	add thirty-eight to forty-five
S4	eighty-three?
T	how did you find out?
S4	I first added the tens, so eighty-three plus for-ty
T	thirty-eight.?
S4	ya (forty.eight)
T	what did you add now?
S4	I added thirty-eight to forty and that would be seventy-eight
T	so you did thirty-eight PLUS forty!? Is?
S4	seventy-eight? And then seventy-eight plus five. That's eighty-three.

S1	I did it, plus, ehm forty-five plus thirty is seventy-five and then, I have- from the eight, I first went to the-, from the eight which were left, I first went to the next tens, and then the last three.
T (09:55)	What is three-hundred five take away eight? What does that mean?
S2	three hundred five minus eight (…) equals two hundred ninety seven (…) Because I did, first I, made from the three hundred minus five (…) from the three hundred and five, I took five away. And then I (? Need to) take three away and then I took them away and then it was down to ninety-seven
T (13:55)	what is half of six hundred fifty?
S5	three hundred twenty-five. I did half of six-hundred and then I did half of fifty. And then I got the answer. Like, I did, I did like half of six hundred which equals three hundred and then I keep three hundred on my side, on my side of the head. And then I do half of fifty which equals twenty-five and then I add twenty-five to the three hundred and then they equal three hundred and twenty five.
T (16:00)	A square has sides four centimeters long. How long are all the sides together?
S5	sixteen
T	how did you find out?
S6	because four times four is sixteen
T	why did you do four times four?
S6	because there were four lines on a square, like, (there, there, there and then, so) one, two, three, and four. And they are four centimetres long, so I did four times four, and, ya, is sixteen.
T	but what do you know about a square that you do four times four? What do you know about squares? That you know that you have to do four times four?
S6	that there are four sides on a square?
T	yes, a rectangle also has four sides. Or a kite also has four sides.
S6	ah, because they're all the same length
T (17:15)	A packet of cheese has a mass of two hundred twenty-five grams. What is the mass of two packets of cheese?
(…)	

S3	four hundred fifty. And I made, ehm, doubled two hundred and then, I got forty, and then I made- doubled- twenty five
T	you got forty?
S3	ehh, four hundred.
T	what did you do to get four hundred?
S3	I doubled the two hundred. and then I got four hundred from- and then I doubled twenty five. So I got fifty- and the answer was four hundred fifty.
S2	or you can say two hundred twenty-five times two!?
T (19:25)	how many centimetres is two point six five meters?
S7	two hundred sixty five!?
T	what?
S7	m, centimetres!?
T	how did you find out?
S7	I knew it
(…)	
S8	because hundred centimetres are one meter and two hundred centimetres are two meters and then, I just added sixty-five centimetres and then, because hundred centimetres are one meter, it's two meters sixty-five.
T	so, we know, that we have two meters and sixty-five centimetres. Because we know after the decimal dot, we have the centimetres. And we know that one meter equals hundred centimetres. So two meters equal- ?
S1	two hundred centimetres
T	and then we still have to add sixty five centimetres
T (22:35)	What is the difference between two hundred eighteen and two hundred fifty? What does it mean? What does it mean, the difference?
(…)	
S2	the answer is thirty-two because, because we wanted to say how much- how many- ehm, is the- how- like, because, because, thirty-two plus two hundred and eighteen is two hundred and fifty, and, ya!
T	how did you find out?
(…)	
S9	I did two hundred fifty minus two hundred eighteen. I did that like, first, I did two hundred fifty minus two hundred is fifty and there is still eighteen left. So I

did first ten forty and, but because it’s eight, I did minus ten again, and then I did plus two again, it’s easier

S2 you could just, at first, add to the next tens. And that would be then the twenty and then add up to fifty.

Class level: **Grade 4**

May 2013

T (00:40) What is a square number?

S1 Numbers, so when you like, one times one, so, number, or, two times two, so numbers that (?), so multiply by themselves.

T yes, a number that I multiply by itself. Why is it called a square number? Can somebody explain me why don't we call it a – I don't know, a multiply it by itself number?

S2 (01:15) because if we have for example, one times one, and then you want to draw it in blocks, you can always make it that it looks like a square. If you have two times two, you can have also that it looks like a square
(…)

T What is the next square number?

S3 four times four

T four times four. That is?

S3 sixteen
(…)

S4 seven times seven?

T (03:00) yes, seven times seven. And that is? (.) wait (.) How far are you? Five times seven already? What is five times seven?

S4 thirty-four?

T thirty-five. And what's six times seven? (.) shall I tell you?

S4 forty three?

T ah, forty-two. And now, seven on top. Is? What is two and seven?

S4 eight?

T ah, one more is eight and when I have two more? So I have seven and when I have two more, then I have? Eight fingers?

S4 (nine?)

T Nine!! So then we have forty nine!! Yeah? Very good Cameron!

T (05:30) What are the squared numbers between ten and fifty? Which square numbers are between ten and fifty?

S2	twenty-five, forty-nine, sixteen, thirty-six, wait-
T	four, five, six, seven
S2	not more.
T	what are the squared numbers between sixty and ninety.
S5	sixty-four!?
T	so this is? sixty-four is which multiplication task?
S5	eight
T	eight times eight. So the next one would be?
S5	eighty-one!
T	eighty-one, that would work out. Ok.
T (07:20)	what are the square numbers between zero and thirty? (.) which square number? Which (?) do you know?
S6	(four?)
T	four! Ok, is it between zero and thirty?
S6	what?
T	yeah, is it between zero and thirty? Is it larger than zero?
S6	yes!
T	Is it smaller than thirty?
S6	yes!
T	then it fits in between! So we have the first one. Is there another one?
S6	yes.
T	which one?
S6	five?
T	is five a square number?
S6	eh, no!

T	square number.
S6	twenty-five?
T	twenty-five. (.) so, this is a square number of what? Of-
S6	of two?
T	two. (.) this is the square number of?
S6	five.
T	five. (..) and is that all?
S6	mmh, no!? (.) sixtee—no
T	no? don't you think so? What is with sixteen? (.) is it larger than zero?
S6	no six- ya! Sixteen!
T	ya? And sixteen is a square number of?
S6	four
T	ya! So it comes in between. four times four. Do we have another one?
S6	two?
T	two is not a square number, even easier
S6	seven? (.) nine?
T	nine!? Ya, but even easier!

T (10:08)	I am thinking of a number. Half my number is eighteen. What is the double of my number?
S7	seventy-two?
T	seventy-two. Why? Why do you say seventy-two? (.) We'll go step by step. So, the first sentence is: half my number is eighteen. What do you do? What is my number then?
S7	it is thirty-six.
T	thirty-six. How did you find out thirty-six? What did you do?
S7	I did twenty plus- eh, ten plus ten and then I did eight plus eight.

T ah, so you doubled the eighteen!? Why did you double? How did you know that you have to double?

S7 because that was the half of – mmh- the number.

T yes, and if it is the half of my number, then I have to do the opposite operation and that is doubling. Perfect. But we're not finished yet because then the second sentence is: what is the double of my number? So what do you have to do?

S7 add thirty-six plus thirty-six.

T good. And how did you calculate it?

S7 ehm, thirty plus thirty is sixty and six plus six is twelve.

T good, and altogether?

S7 it is seventy-two.

T (14:28) What is a factor?

S8 I think something that fits in plus the- oh no- that fits in times! (…) so, ehm, factor times factor equals product.
(…)
T twelve and eighteen. how can you find out common factors, or factors both numbers have the same? Do you have an idea?

S3 No

T No? Do you know the time table facts? Which two number we can multiply that we get twelve?

S3 six! Six times two!

T oh! We have another one?
(…)
S3 four times three!

T yes

T (33:45) If we would now introduce the sixteen and the thirty-six – what could we say about these two numbers?

S3 they both have the six in there.

T (they're) the digit six. Yes so there are the same amount of units

S2 you can also say both of them have tens in them

T oh, yes

(…)

Both have six units, that is what they have in common. What do they have in common as well?

S1 they're both equal

T they're both equal?

S1 ah, no I mean (.) even!

T so both are even number! (…) and? What as well?

S3 They both got the factors of one, two, and four.

T what do they have?

S3 they are multiplied by one, two, and four

T ah, ok! So both are products of one, two and four. But something else! What did we discuss today?

S9 I think they're both square numbers

T ah, thank you, look: they're both square numbers

Class level: Grade 5

August 2013

T (02:55)	now, we have the task five point seven plus mmh equals eight. How can we calculate this? Does somebody have a strategy?
S1	you first say five plus eight? Wait, is that? Ya, first you look what- uhm – how much it is till ten, from seven,
T	ah, so five seven, we want to know up to? What is the next one? After five point seven, we have?
S1	six.
T	so if you have the number line, it looks like that: five point seven and here is six. So how many hops to the six!? And how many are there?
S1	three.
T	three?
S1	ya, eh, three, eh, zero –
T	zero point three, ok. Good, so, because here is five point eight, five point nine, and as you can see, zero point one, zero point one, zero point one. So we need three times zero point one to fill up to six. And now, what do we do then, in the second step?
S1	then we have to look how much from six to eight. That's two. And then you have to two and zero point three.
T	and together two and zero point three?
S1	two point three.
T	there is also another strategy. Did somebody use the other strategy or did everybody fill up? (…)
(05:05)	did somebody subtract?
S2	ya, I think I (?) eight minus five point seven
T	yes. So you can fill up, that you can look, ok how many decimals do I need to fill up to the next unit and how many units do I need!? Or, I can see ok, this is a addition, to find out a missing number is that I subtract, that I say how much of, how many is it when I have eight minus five point seven. Somebody know hoe to calculate it this way? If you would subtract?
S3	I would do it, first, ehm,

T how many would you take away first?

S3 first, I would take eight, ehm, so, zero point eight away

T you can't. look, the five is five plus zero point seven. So what would you take away first?

S3 ehm, the half of eight. No, five.

T five! Ya!? How many are left over?

S3 zero point seven

T you take eight minus five is?

S3 (.) oh

T it's too easy, I see! So, now three is left over and then now, we take zero point seven away

T (07:25) now, we have the next magic word: halving. What is halving? What does that mean? Or to halve?

S4 (it's what we do with a number) when we do like that half of it

T really? And what is half?

S4 mmh

(…)

T if I would have four candies, how would you halve them? What would you do?

S4 I would take two of them away (?) (and have two!)

T two what?

S4 two candies!

T two candies. And how many parts do you have?

S4 two?

T and what is- what do you know about the parts? They are?

S4 all together four?

T yes, all together four, but if you would compare the two halves, then they are? If you look at the amount?

S4 I don't know?

T	what are they?
S5	they would be half.
T	yes, that's true other wise it wouldn't be half (halve?) But if you would compare them: you have four candies and in one group you have two and the other group, you have two as well. Now you compare: this half with this one. What can you say about the two halves?
S5	they are the same.
T	ah! They are the same amount! So, if they say halving or to halve, that means: divide a number into – now very important – two equal
S6	parts!
T	parts, ya!?
S3 (09:56)	oh, the funny thing is: two plus two equals four and also two times two equals four!
T	mmh, it's the inverse operation. Halving is the opposite of doubling. And doubling, we had last time, right!? You double with times two and when we halve then we divide it, by two. (…) So, we have forty, then it goes down to twenty, then it goes down to ten. Then we have ten and then we have to halve it. What is half of ten?
S7	five.
T	five, ok. (..) and then, it says, halve this half again into two halves. And it is?
S8	two point five.
T	yes. (…) can you remember what I said what you can do when you are not sure how two halve full numbers? Or whole numbers? Did I tell you how? What is the easiest way to help yourself in a test?
S9	can I say how I do it? (…) I think, (I just take) four, so two and then (?) I just the half of, ehm
T	so you knew one was left over
S9	ya
T	and then you
S9	I halved it.
T	ya

S9	and then it's two point five.
T	yes. So you put the two halves together. I would always use money. Is that easy to halve five euros? Mmh, ok, so how (…) if I would halve seven euros, how much money would I have then?
S10	three point five
T	why?
S11	because three plus three is six and then one (?)
T	yes, so six and one, you divide the six into three and three and the one euro that is left over you divide into fifty cents. (…)
(13:30)	The line shows a pair of numbers that add to one. (…) Draw four more lines to join numbers that add up to one. So what did we think about when we add up one – a number with one decimal place? What did we do to fill up? I want to add up to one, what do I always have to think about?
S5	that seven plus three equals ten?
T	yes, so how much do you have to add?
S5	zero point three
T	yes, because seven and three add up to ten. Now, we have something like this: zero point seven five and we want to add up to one. What do we have to think about? Can you remember? How do I know how many we have to add?
S3	if you we're doing it, that instead of zero point seventy-five, we're doing seventy-five plus what is hundred. And that would be then zero point twenty-five
T	two five. Zero point two five.
S3	zero point two five. Because zero point seven five wouldn't equal one if you would plus zero point three five, because then it would be one point one
T (16:15)	so if we have zero point zero five, how much do I have to – or how many do I have to add that I get one?
T (25: 10)	so, what do we do to find out the left number? What do we do? The number line: positive and negative numbers. How was it when I go to the left? How do the numbers look like? What happens? When I go to the right, how do the numbers look like? What do you do? To find out this number?
S12	mmh, minus three?

T	ok, and what number is it?
S12	mmh, minus five!?
T	and now, what do I do on the other side? How do I find that out?
S13	negative two plus two is zero plus one is one.
T	ya, so we first go up to the zero, so that would be plus two. And we want to go three, so we just have to add another one
T (26:45)	short revision of negative numbers. Which numbers are larger? Look, I have: minus one million and on the other side I have five. Which number is larger?
S13	five
T	why? But the million is far way longer
S13 (27:33)	(?) if you go behind zero it is smaller, and before zero is bigger
T	yes. So all the numbers that are in the right side of the zero are – how do we call these numbers? Do you remember? What's the name of these numbers? P- posi-
S13	positive numbers
T	yes, if we look at the number line, everything to the right of the zero are positive and everything left from the zero is negative. And we said the more we go to the left, the smaller the numbers are, ya!? So even if this is a huge number, it seems to be a huge number but negative thousand – million, is a really really small number compared to five
S6	what is if the minus would be gone?
T	what if is the minus would be gone?
S6	(?)
T	What is if I have minus one million and I have minus five?
S14	minus five is bigger
T	ya, so if I compare two negatives, the number that is nearest to the zero is the larger number. So of course when I would look at the number line, here is the zero, here is minus five and way here is minus one million. So one million. Really far way from the zero. And the minus five is very near to the zero and that's why minus five is more. And positive number, never ever forget, are always larger than negative numbers.

T (32:15) What is subtract? The word means what?

S6 minus

T so, we have fifteen point four minus – no! They say subtract fifteen point four from twenty-one point one. How do I calculate it?

S14 first I would subtract four minus one and that is three.

T Wait a minute, here, you really have to be careful what you read because it is subtract fifteen point four from twenty-one point one so which is the number we take away? So which number is the amount that we have? At the beginning?

S5 (?)

T no, we have two numbers: number fifteen point four and number twenty-one point. and we want to take away. but what is the amount we have at the beginning before we start subtracting? We have to have an amount of – is it this number or is it that number?

S15 (?) eh, the other

T this one!? Why?

(.) What do you need if you subtract something? Is it good to have a small number at the beginning? No! ya? So in this case, we always, we take the larger one in this case and it says here: from! You take away from this twenty-one point, ya? So we first of all have twenty-one point and then we take away fifteen point four, ok?

[on board 168 / 6]

S1 (08:15) first, you look ten times six. And that is sixty. And then you have to look how much it fits in one hundred sixty-eight. and that's twice. And then you have to look at the rest.

T the remainder, ya!?

S1 yes, And that is forty eight. And then forty-eight divided by six is (.) eight.

T good, and how often does it fit into the one hundred sixty-eight in total?

S1 it fits twenty eight times.

T	(…) she saw that one hundred sixty-eight is definitely more than ten times six. So that's why she tried out how many sixties fits into this hundred number. And in this case it is twenty times. And then she saw: ah this left-over here, the forty-eight, is under, or beneath ten times six and then she knew it is eight times and then she counted how much- or she added how often the six fits into the one hundred sixty-eight.
T (12:00)	What's an improper fraction? What's the definition of the improper fraction?
S2	the denominator is always, or, more or less than- the numerator is always less than the denominator
T	ok, first of all, where is the numerator?
S2	the numerator is beneath
T	ah, no you swapped it. So first of all, the numerator is up on top and the denominator is in the bottom and then your definition is correct. Say it again
S2	the numerator is always higher than the denominator.
T	so the number in the numerator is always larger or equal to the number in the denominator. And can you remember: how big is a fraction? If we would compare it to a certain number? Are- is a fraction a number, first of all?
Stds	no, yes, no, yes
T	What do you think? Who thinks yes? (…) seven. And who thinks a fraction is not a number? – ok, and the others don't know!? So, a fraction is a number, it just looks weird. It is a number, the new thing is just that we have two numbers over each other with this funny line. But where do I find fractions on a number line? Can you remember? When we would draw?
S2	between zero and one
T	yes, we find them between zero and one. Here are the fractions and here are the proper fractions. (…) so between zero and one we find the fractions. Because fractions are numbers, they just look weird. And they are smaller than one. They are tiny numbers, they are small small numbers. Ok, and the proper fractions are always smaller than one and the improper fraction, because it has more parts than the one has been divided in, is, then,
Stds	over (?)
T	larger than one, yes. So improper fractions are larger or equal, of course, to one. That's the definition. So we can recognize improper fractions when the numerator has equal parts to the denominator or more. (…)
(18:50)	ok, where do I find seven over four? On this number line?

S1	(at) the one, and then at the end of one.
T	at the end of one?
S1	ya, not number two but behind the number to this
T	how do you recognize that there is a one in it?
S1	because it's , because seven, because it's less than
T	what would be one? How many parts do I need to get a one?
S1	four
T	ah, so first of all, if I have four parts, then I already have a one. So, and how many parts are then left over?
S1	three.
T	three.
S1	that's why
T	so you can see when I have four, then I have a one, so I'm already here. I can see the ones in between are divided into four equal parts and these three are left over. So, I hop to the one and then I then I have three parts and here is seven over four. (…)
(21:05)	Where is four over two on the number line? (.) how many parts do you need? Is this a proper fraction or an improper fraction?
S3	at the two?
T	why?
S3	I don't know why. (it's just in my head)
T	well, maybe, how many parts do you need to get to one?
S3	to the one? (You need one?)
T	how many parts? What tells you how many parts you need?
S3	three?
T	why are you guessing? Look, do we have a three in this fraction? No! so, what tells you how many parts the one has been divided in? the numerator or the denominator?
S3	mmh

T	ah that's not funny. The number up on top or the number in beneath, beneath it?
S3	the number underneath?
T	aha. In the bottom, so the denominator. So, there, we have a two: so how many parts do we need to get to one?
S3	two?
T	ah! See, you don't have to guess. It stands there. Ya!? (…) When we need two parts to get a one, how many parts do you need to get to one? How many parts do have to have in the denominator, eh in the numerator, to have a one?
S3	two?
T	ah, see you don't have to guess again! Now, look, what is left over? This is the one! Now, what is left over? How many parts? (.) You had four, two out, so how many left over?
S3	two?
T	ah, two! So, now, we have another two over two, this is a one, and when we add up the two ones, we have a- ?
S3	two.
T	and that is the answer. So four over two is- ?
Stds	two!
T	two. Now the easiest one. Three over three, that's the easiest one! Three over three!
S4	no, no, no! I had a question!
T	first, three over three. Three over three is equal to? (.) Look, in how many parts does the one has been divided in? into?
S4	three?
T	ah! And how many parts do you have? Come on, it stands up there!
S4	three.
T	ah, so if the one has been divided into three equal parts and all three, you have. What do you have?
S4	three.

T	three? So, you have one pizza. You divide it into three pieces. You ate all three pieces. How many pizzas did you eat?
S4	one.
T	ah, one! It's one! The parts are equal here, so it's one.

T (27:50)	Do you know what this sign means?
S5	so like five time ni-
T	- wait: what's the meaning of this sign?
S5	like is or!?
T	ya, is, or? Do you know it? The name of the sign?
S6	equal.
T	equal.
S5	ah, the name! I though what does it mean!?
T	so, equal. Equal to, actually. No, what does that mean for both sides? When I see this sign?
Stds	(?)
T	No, what does that mean on both sides, when it has to be equal?
S6	equal means that on both sides, it is the same.
T	ya, same amount. So, on both sides, the same amount. So we want to have the same amount on both sides. So, if we now look at this side, how much, how many do we have?
S5	a hundred?
T	ah, so ten times ten is a hundred, so on this side, we have a hundred. So we have the equal sign, so how many do we want to have on this side?
S5	zero, no, ten, ten, a hundred!
T	so we want to have hundred on this side as well. Otherwise, the sign is not allowed to be there where it is. So we already have hundred and now they say you have to take something away, and we know it has to stay hundred. Tricky
S5	zero

T	zero.
T (33:00)	What you have to do to find out is: In how many parts has the hundred been divided in. Look, here is one part, here, here, and here. So you have to imagine: we have one hundred millilitres and we put- we divide the one hundred millilitres into four smaller jugs. How much, many, millilitres do we have in one jug?
S7	ehm, hundred
T	uhu, we have one hundred and I divide it into four equal jugs
S8	twenty-five
T	perfect. because hundred divided by four is twenty-five. One of the important time table tasks. So, what here is that we have two hundred and here is twenty five. So we are, at the moment, at two hundred twenty-five millilitres. (…) so now, on top of the two hundred twenty-five, we add water, one hundred fifty millilitres. How much water do we have at the end? When we add (.) one hundred fifty millilitres. How would you add it?
S7	I would count till (?)
T	how? So one hundred fifty, so this would be how much? Twenty five, ok. And if I go two hops, then I already have?
S7	it's fifty?
T	it's fifty ya!? So, here we already have fifty, ok!? And we want to add now one hundred fifty. So, how many are missing now? How many millilitres
S7	hundred!
T	hundred. What do you do?
S7	(?)
T	Wait a minute, two jugs is fifty. So from here to here, it's twenty five. Another twenty-five is fifty. We're here. If we hop another one, then it's again twenty-five.
S7	(and then another which is a hundred and then again one and that's)
T	we're by one hundred twenty-five
S7	(and another one)
T	So the next water level would be here (…) we add one hundred and fifty (…) What would now be the answer?

S2	three hundred seventy-five.
T (38:30)	Next magic word was equivalent fractions. What are equivalent fractions?
Stds	it's like if the numerator is smaller than the-
T	uhu, that has nothing to do with proper fractions or improper fractions. That's now if we- so equivalent fractions is when we start comparing fractions. So in this case here, we look at these three fractions: a half, two over four, three over six, they all look different but what do we know about these three fractions when we compare them?
Stds	They always go to (two) and I think they're also the same
T	yes, they are the same value. So, equivalent fractions, in this case, all three fractions here have the same value. They just look different. The fractions are totally different but they are still the same. (…)You need to get used to (…) Now, how do I recognize that they are the same value?
S1	because they're always the half of the- of the number- of the denominator.
T	yes. But it's not always like that. How would you now recognize that these two fractions are the same as well?
S1	because three and, three times five is fifteen!
T	aha, and five times five is?
S1	twenty five.
T	See, so this is how we can recognize. Because the numerator and the denominator have been multiplied by the same number, so in this case they have been multiplied by five. (…) So equivalent fractions, we remember, we can recognize them by comparing the numerators and the denominators and when they have been multiplied by the same number, then they are equal. Ok!? (…) you have three over four and now you ad to find the equivalent fraction that has a six in the numerator. So how does the denominator look like?
S7	(It's always the ?)
T	ya, but which number do we have then in the denominator?
S7	we have eight?
T	yes! How do you know that you have to double it?

S7	because the other one has been doubled?
T	yes! And that's why it is a eight down here. Then we had four over five, and we have another and down we have fifteen and now: which number comes in the numerator? (.) What happened with the five here?
S6	it was timesed by three.
T	yes! So which do-
S6	-twelve.
T	twelve! Ok, and the last one was two over five equals mmh twenty-five
S9	ten?
T	sure? (.) no, it's correct.

T (47:10)	next magic word: perimeter! For example: the perimeter of a rectangle. What does that mean?
S10	(all the sides and how long they are)
T	yes, and what do we do with all sides? We-?
S10	we, ehm, we do, we calculate how long they are
T	what do we calculate? Plus, minus, times, divided?
S10	(one is – uh, the left is sixteen centimetres long, on the side is two centimetre long, we need to calculate
T	What does the sixteen centimetres mean?
S10	the sixteen, it is (the part) in the length of the rectangle
T	yes. So that means when we start here and we walk one way around, ya? This is the perimeter. So we- and the word is add- we add up all sides.

(…)

S11	and then, if one side is two centimetres long, and fourteen centimetres are left, one side needs to be two centimetres long, and then there are twelve left and (?) two centimeters, and
T	why does one side have to be two centimetres?

S11	because one side has - is already two centimetres
T	and why rectangle, is, that the opposite sides are the same, so that's why this side has to be two centimetres as well. So, we had sixteen minus two is fourteen minus two, and twelve are left over.
S11	and then the half of twelve is six. So then the left over sides are six
T	yes.! The opposite sides have to be six. Because the have to be equal as well.
Stds	six?
T	ya! Because we have to divide the twelve into two equal sides.

T (52:10)	next magic word: square numbers. What is a square number? Do you still know that?
S12	it's like, one, times two times two
T	three times three? Four times four?
S12	ya, like
T	so what do you do with the numbers, you-?
S12	(find them) no, you times them!
T	yeah, by?
S12	by (?) square numbers!
T	yes, but to get a square number, what do you have to do? You take a number, what do you do with this number? You multiply it by?
S13	itself?
T	yes! Itself. So to square a number, you have to multiply – and you can take really any number, it doesn't matter, you can take the hundred, you can take the ninety-nine, the seven, it doesn't matter- any number, and the clue is, here comes, by itself. For example, one times one, (…). And the outcome is the square number. These are the square numbers. Why did they give these numbers the name square?
S6	because if you make those numbers like in blocks or so, and put them altogether, they already look like squares.
T	yes. (…) so if I would show these multiplication fact, it always look like a square and that's why (…)

S1 (19:15)	what if there would be zero zero zero zero, and down is uh, six five four
Stds	then it would be six five four
T	doesn't matter
S1	but then, if there is zero zero zero zero, then there, uhm, that wouldn't be a number.
Stds	zero is a number
S1	ya, but
T	no, what you mean is if
S1	No, just, without the seven, ya, like that
S2	that doesn't exist
T	that's nothing
S1	but it's zero
S3	no, it's nothing, it's just nothing
(?) (…)	
T	but if we would have it like that, could we solve it?
Stds	yes, yes, yes Negative numbers
T	aha, which negative number would it be? (…) It would be like zero minus three hundred seventy-five and that is then negative three hundred seventy-five (…) But this is not a number. Nobody would write that.

T (20:50)	how would you calculate this here?
S1 (21:30)	I would do seven minus six and, minus zero, is seven. Six minus zero is six. Two minus three doesn't work, so I have to put one, a one from, so, put it, twelve minus three is nine. And two minus five doesn't work, so I have to (take one) to the two, and twelve minus five is seven and nothing minus five is five
T	nothing minus five is five?
S1	nothing minus five is nothing!

T	nothing minus five is nothing!? What's the problem now?
S1	that you don't have anything to add to the five, so, uh, subtract
T	subtract
S1	ya,
T	so, what's the problem now? Can you solve it?
S1	no
T	no, you can't solve. Why can't you solve it?
S1	because there is nothing to subtract from the five
T	ya, because look at this number and look at that number! Can you subtract this number from this number? First of all? What do you see if you would compare these two numbers? What do you see? (.) look, which number is larger? This one or that one?
S1	the one, that one
T	the higher one. So, can you subtract a bigger one?
S1	no?
T	no!
Stds	you can
T	but not in this method. So sometimes, it is a good idea, before you start calculating, to look at the numbers, because sometimes, it's not possible. So in this case, is, that you cannot subtract it because we, as you already said, we have nothing to subtract. (…) it isn't possible because you don't have enough – then you don't even have to start. (…) we didn't have enough to take away

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T (02:30)	Now the problem is, here, it is very easy because we have a two digit number and we multiply it by a one digit number. Easy as well is to multiply a three digit number by a one digit number (…) of course, the higher we get, the longer you are at school, the larger the numbers will be and then it could be that you have to multiply without a calculator something like this here: twenty-three thousand six hundred twenty-one times twenty-seven. And if you now start to use this method, that takes ages. And that's why we are going to look for a method that gives us the possibility to calculate, later on such large numbers but we'll start with smaller numbers to understand the method, that we can calculate such large numbers in a very fast way.

S1 (32:50)	… is fifty-four?
T2	no.
S1	no? ok, then I have to, first, five times seven, no, five times eight is, no, wait, thirty- no forty!
T2	mmh
S1	plus fourteen
T2	why fourteen?
S1	because- no, wait. So, forty, forty, and now there's fourteen left over, no!
T2	so, what have you been doing so far? You did five times eight
S1	ja!
T2	and you want to do?
S1	seven times eight
T2	ok, so what's left?
S1	seven?
T2	no you did five times eight and you want to do seven times eight. So, how many eights do you want?
S1	uh, seven!?
T2	you want seven eights, right!?
S1	ya!
T2	and you got, how many eights?
S1	five
T2	and so, what do you still need?
S1	three! Uh, so three eights, no
T2	if you have five eights and you want seven eights
S1	I have five eights and I want seven – so I only need two more eights
T2	two more eights, exactly. And what is that?

S1 so and that altogether is

T2 what are two eights, first of all?

S1 two eights are sixteen! Now, I'll just sixteen plus, so it's five, so it's fifty-six.

T2 exactly!

S1 and now, I'll do seven times five is forty, no, thirty-five. So, and then fifty times seven is three- uh, three hundred and fifty.

S1 (36:00) eight times five equals forty and twenty-four are left over

T2 why?

S1 because three eights are left over and that are, equals twenty-four. And then, altogether, sixty-four.

S1 (04:45) and now you plus four plus zero plus zero plus zero is four. Then you plus two plus eight plus zero plus zero is ten and that's why we put a zero there (…) and then you make instead of the two a three. And three plus zero plus zero is three. And two plus eight is zero and then you put a one in the – uh

T next column!? Ya? And then, is here?

S1 one.

T perfect. What's the name of the number?

S1 million-uh- ten thousand.

T ah, so again: units, tens, hundreds, comma, thousands, ten thousands.

T (08:00) for example: [writes on board: 538 x 6]

S1 eight times six

T six times eight, ya
(.)
What is five times eight?

S1 five times eight is (.) forty

T and then six times eight, has eight more, is?

S1 forty-eight!?

T perfect! So forty-eight, now: what do we do?

S1 ehm, we take the eight, from the forty-eight, and put a small four over the eight, and

T ah, so we put the, keep the four in mind and the eight?

S1 the eight is for the units

T yes, we write it in the units. Ok, next step!

S1 six times three is eighteen, so, we take, then we need to take the eight from the eighteen and

T ahah

S1 and add up with the four

T yes, first, add the four because we have the four in mind, ya? So that's why I always have the fingers because as soon as I add the four, I put my fingers away. So I know it's gone. So that's twenty-two. Now, what do we do?

S1 now we take two to the, (there, to the tens, column) and put the, uh, remember the one, uh the two!

T ok, last step?

S1 Then you, six times five equals thirty. And then we need to add up with the two. And that is thirty-two. And then we need to take the two to the hundreds column and the three in the thousand

T perfect.

T (15:10) First of all, we look at the first digit of the number, the two. And we think about, do we, does the four fit into the two? Can we divide the two into four parts?

S2 no

T First of all: two divided by four, does that work out?

S2 no, yes, no

T no, ya, if you want to have fractions, but we're not thinking, ya, without remainder.

	That means that two divided by four doesn't work out. So that's why we have to take the first two digits. And then we have a twenty-three. Now, does the four fit into the twenty-three?
Stds	yes, yes
T	So, this is the thought: twenty-three divided by four. How often?
	five times
T	five times. Because twenty divided by four
S3	is five
T	is five. But
S3	there are three left, so we have to write remainder three
T	aha! Ok, so, this is the (talk). We have to know, ok, I divide the twenty-three in, by four. I know the four fits five times into the twenty. So that's why I write down the five up on top, here. Five times. So, up on top the answer, we write above the number we divide. ok? Now, again, the thought is: how often does the four fit into the twenty-three? Five times. Now, second thought is: because five times four is twenty, that's what we write underneath. Then we draw a line and we subtract twenty-three minus twenty. And we write the remainder underneath. Ok, we're gonna practise that over and over.
S4	should we write it under the zero or?
T	under the zero. Because it's a three and in this case, it's a unit so that's why we have to, ok! Before we now continue, we have to take down the six. So the six is now, well we don't take the six but the six, we now write beside the three. Like this, so that we have a bigger number. And then, the division task is then? Thirty-six divided by four equals?
S3	is (.) nine.
T	yes. Is there a remainder?
Stds	no. no
T	ok, so, now, this looks like this: thirty-six divided by four, so, or, the four fits nine times, the nine comes here up on top on the line, nine times into the thirty-six. Because four times nine is thirty-six. That's what we write underneath. We draw a line and subtract. Thirty-six minus thirty-six is? Zero! We're finished. We'll do it again.
S5	a long method for the result

T actually not.

(…)

T (21:10) If we look at our task, the first step is: how often does mmh, in this case four, ya!?, how often does mmh fit into mmh. So that means if you read it, it is how often does four fit into twenty-three.

(…)

S6 into? What?

T ya, into mmh. Because every questions has different numbers.

S6 ya, but what is it right now?

T so, here it would be twenty-three.

S6 ok

T but you don't. so: Where do we write down the answer? Where? (…) So, how often does mmh fit into the huh – then we said five times. Where did we write down the five? That's the answer. Where did we

S7 there: up

T yes, here, so the answer, write the answer on the line above.
Second step:, so, you see it again: how often does the four fit into the twenty-three? Five times. And we wrote the answer on the line above.
(.)
Now, the second step is: because (…) four times five is twenty, ya!? So, because mmh times mmh equals mmh. That's the multiplication fact. Where do we write down the answer? Look! We had this: because four times five is twenty. Now this number, where did we write down the twenty?

S3 we wrote the twenty beneath? To the twenty-three?

T yes, so under the number we divided. So: write the answer under the number we divided. As we can see here: because four times five is twenty. (.) below! Not under. Sorry, below, that was the word.
(…)
Ok, now third step.

S8 should we put the numbers in the gaps?

T no, you don't have to. The gaps are because, the gaps are later on, because for every task, you'll always have different numbers. That's why. You'll see. We'll do one together then. Ok, now, last step is. What did we do the? Can you remember? So it was-? What did we do?
So, first of all, it was how often does the four fit into the twenty-three.? We said five times, the answer up above, uhm, on top. Number two: because four times five is twenty, we wrote the number beneath the twenty-three. Or below the twenty-three. What do we have to do now? What was the step? Third step?

S8	write the, write the remainder, remainding, remainding numbers?
T	the remainder?
S8	yes. The remainder underneath the answer
T (29:35)	ok, so, here, we drew a line, ya!? So: draw a line!, ya, so we just think of line. Draw a line. Then we know what to do. And subtract! And then: start with one. And we start with the beginning on again.
T (31:05)	[writes: 165 divided by 5] ok, now, we'll start. And always keep the sentences in mind. So: first of all we start: how often, with sentence number one, how often does five fit into the? Which number? Sixteen. So, how often does five fit into sixteen? Because of course, one doesn't work out, so that's why we have to take the fifteen, ya? How often?
S9	three (remainder one)
T	so where do we write down the three now? Now, we have the sentence: write the answer on the line above. So we write the three above, ok? So, now, step number one finished. Step number two: because five times three is?
S10	fifteen!?
T	fifteen. Where do we write down the answer?
S10	uh, below?
T	the number we divided, ya!? So: fifteen. So we write down the fifteen under the sixteen. Like this! Ok, now: number three: draw a line, zack, and subtract! So sixteen minus fifteen, L already said it: remainder?
S9	one.
T	one. What do we have to do now? Are we finished yet?
S10	no. no
T	what do we do now? What's the next step?
S10	do the five, also: runter
T	we take it down, ya, or we pull it down, or So: the five here is left over, this one, we pull down, like down. Because this is a remainder, ten is left over, we have to take down the five, so that we have: fifteen! See it? Like that. Ok? And then, we start from number one, again! How often does five fit into fifteen?

S11	three
T	where do we write down the three? (.) What does it say? Above! Ya?
S11	above
T	so: above. Here. We took down the five here, so the three fits above. Here, we write it down. Ok? So, number two: because five times three is?
S11	fifteen?
T	fifteen. Where do we write down the fifteen
S1	uh, underneath? Below?
T	yes, below the fifteen. Ya? Like that! Now, number three comes: draw a line, zack, and subtract. Fifteen minus fifteen is?
S12	zero.
T	zack, finished we are.
S13	(what is) the answer!?
T	the answer is this here: look! So, how often does the five now fit into the six hundred – oh very good, see, I forgot this step totally! So: how often now does the five fit into the sixty-five? How often now? Where is the answer? Where is the answer now?
S3	uhm, on top, the thirty-three.
T	this here! This is the answer: thirty-three times.
S14	it makes more sense when it should be (at the end)
Sb	so here? (up) at the line, is the answer!?
T	up here. Yes.
S14	yes, that's the answer!?
T	that's the answer.
S15	it makes no sense. I (?)
T	it makes sense!
S15	no, because the result is up there and it should be down here, it shouldn't!

T	no, you have to think like this, look: this is the task: so the five divides the hundred sixty-five. So that's how you have to think. Now, this here is your calculation to find out how often the five fits into the hundred sixty-five. The answer is up on top. It's a little bit weird because it was always, normally the answer goes always underneath and now it's up above. That's why the division is so complicated because you have so many steps.
T (37:05)	so, the seven divides the five hundred eighteen. Again, we look at the sentence: how often does seven fit into the?
S3	fifty-one.
T	yes! So: how often does seven fit into the fifty-one. How often?
S16	seven
T	seven times. Where do we write the seven?
S17	on the line above!?
T	yes! Here: above. So: above: seven. Now, we go to number two: Because ? how does the sentence- complete the sentence! Because?
S17	seven times seven equals, uh (.) seven times seven, uh, forty-nine.
T	ok, perfect. Where do we write the forty-nine, now?
S17	uhm, underneath. (below)
T	yes, below. Ya? In this case: below. Forty-nine. Now, this is the easiest one: step three: draw a line and subtract! Fifty-one minus forty-nine is?
S18	fifty-one minus what?
T	you see it!
S19	forty-one!
S18	ya, is, uhm, (forty (?) minus (?)) no, is,
T	it's two. Ok, what is the next step? What do we do now?
S7	we put the eight down, next to the two.
T	perfect. So: the eight down next to, or beside, the two. And thus, we have a number again: and, what do we do? We go up to number
S20	one
T	one. So: complete the sentence: how often does?

S9	the seven (fit into) twenty-eight
T	perfect. How often?
S9	uhm (.) four?
T	ah. Where do we write the answer?
S9	uh, up
T	above. Ya? Here, look: above. So, the four comes up on top. Four. Now, step number two:
S21	we already have the answer
S22	ya.
T	perfect, but we have to complete it. Even if we already have the answer, we have to complete it, because:
S23	seven times four
T	is?
S23	twenty-eight?
T	and where do we write down the twenty-eight?
S23	under the twenty-eight?
T	ya. Very good. Then number three: draw a line, subtract, and what is the remainder?
S24	zero!?
T	zero.
Std	it's always zero!
T	it should be. It could be also later on, that we have a remainder. Now, but at the moment we don't have a remainder.
S25	good.
T	and what is the answer now? How often does it fit?
S9	seventy-four!
T	seventy-four times.

T2 (48:15)	you said, yes of course, six fits into the fifty-one six times, but it fits in there even more often than that. How often does six fit into fifty-one?
S26	(.) nine times
T2	what is nine times six?
S26	nine times six is, uhm, fifty-four?
T2	so does it fit in there nine times?
S26	mhmh
T2	no!? so how often does it fit in?
S26	eight times!?
T2	eights times. Exactly. So that's very important. If you look at this number here: the six, you have to, you have to find out how often does it fit in there and the, the largest, largest, uhm how often, you have to take. So: just write it down again! So, lets start again: how often does the six fit into the fifty-one?
S26	eight times.
T2	where do we write it? Up on top. Because: six times eight is?
S26	fifty-one?
T2	no what is six times eight?
S26	fifty?
T2	(.) what is five times eight?
S26	(.) forty-two, no
T2	no: what is five times eight?
S26	five times, five times eight is, five times eight is forty
T2	ok, and what is six times eight?
S26	forty-eight?
T2	very good. And where to you write the forty-eight?
(?)	
	And now you draw the line. Ok, and now you? What do you do now?

S26	subtract!?
T2	subtract. Which from which?
S26	fifty-one and forty-eight?
T2	fifty-one and forty-eight, exactly. And that is? (.) and you do it just like here. Just like in your long subtraction. You subtract this from that. So you do fifty-one minus forty-eight. And that is? (…)What do you do now? very good, you got the three, as a remainder. (.) does six fit into the three?
S26	no!?
T2	no. so you got to fill it up. What do you have left from your problem? That is? What do you have left?
S26	so I have to do this minus this?
T2	no no, wait, first answer my question: what do you have left from your original problem?
S26	the six?
T2	six, so that's the one that you take down. Ok, and now: what do you do? Wait, what is that? Why do you do that? Where do you take the (two?) from? Fifty-one minus forty-eight is just three.

T (00:15)	B. multiplied seven hundred nineteen by nine. Now please, everybody remember because we're now doing the short form, that was new. So how did you do it?
S1	I did first nine times nine is eighty-one. I wrote the one under the nine and I wrote the eight by the one
T	ya, that we keep that in mind.
S1	and then I did nine times one, is nine, plus eight is seventeen
T	is seventeen, ok. (…) so what did you do then?
S1	then, I wrote the seven underneath and I wrote the one by the seven and then I just nine times seven is sixty-three plus one is sixty-four.
T	and then where do you write it?
S1	I just wrote sixty-four!?
T	like that!?

S1	ya.
T	ok, very good.
S2	nine hundred eighty-one times eight. (…) so, first I did eight times one is eight so I wrote the eight down in the units, then I did eight times eight is sixty-four, and then uhm I wrote the four under the tens and kept the sixty in mind. And then I did eight times nine is seventy-two, that means, I added it, so I added two plus six is eight and then I wrote a eight with the hundred and a seven right in front
S3 (03:40)	four hundred ten times four. And I, uhm, zero times four
T	ok, four times zero, is?
S3	uh, zero!?
T	ya, where did you write it down?
S3	under the zero!?
T	ya.
S3	and then, and then the one times four
T	ok, four times one
S3	is four, and you put the four under the ten(s). and then four times four is (.) sixteen?
T	sixteen.
S3	and then you put the sixteen-
T	in front, ok.
T (05:05)	we have three hundred seventy-two times four. How would you do it now?
S4	two times four is eight.
T	ya, so four times two is eight.
S4	then (there) underneath. Four times seven is (.) twenty-eight. We write down the two in front of the eight
T	ok, which number do you keep in mind? The eight or the two?

S4	the eight
T	no, no, no. the one in front. So always the last digit, we write down, ya, down here!? So in this case, it is the eight. And the one in front, the two, we keep in mind. Ya!? Ok. So, and then the last step would be?
S4	four times three is twelve?
T	ya. Is twelve-? And then what do we have to do?
S4	add the, uhm, plus two? Is fourteen?
T	fourteen. And then?
S4	and then you write down (that in front)

T (07:55)	seven hundred seventy-four divided by nine. So: keep the sentences! Say how it works out
S5	so, we have to look at the seventy-seven at the beginning. And then we have to (think) how much nine fits in there.
T	so it's seventy-seven divided by nine.
S5	and it fits eight times, and the remainder-
T	ok, so first of all: where do we write down this?
S5	the eight, we write on top
T	on top, ok.
S5	and
T	now what's the second step?
S5	then you do nine times eight, and that is seventy-two.
T	where do we write down seventy-two?
S5	you write it under the seventy-seven
T	yes, ok!
S5	and then you subtract seventy-seven and seventy-two and that is five.
T	ok

S5	and then you start over, ja, ja, and then you put the four down, so fifty-four. Then you start over and look how much nine fits in fifty-four and that is six times
T	six times!? Ok, where do we write down the six?
S5	up there
T	Up on top
S5	ya, and then you (solve), and then you, ah ya: then you do nine times six is fifty-two? Uh, fifty-four! And that equals, and then fifty-four minus fifty-four is zero
T	ok
S5	and then the answer is eighty six.
S4 (09:55)	why is there a zero every time you (solve a problem)? What is, why is there a zero?
T	why is there a zero? Because you subtract fifty-four minus fifty-four. And how many is left over?
S4	zero
T	and then you're finished, ya, that's why.
T (10:30)	four hundred twenty-seven divided by seven
S5	so: first I do: how often does seven fit into fifty-two!?
T	yes, so forty-two divided by seven.
S5	is (.) six?
T	oh. So forty-two divided by seven is six. Where do we write down the six?
S5	under the two!?
T	uhu
S5	no, under the fou- uh, up!
T	up, ok.
S5	then we do: because seven times six equals forty-two. So we write forty-two, so under the forty-two. Then, we do a line and we put the seven down-
T	but first: subtract!

S5	ah ya, so then it's forty-two minus forty-two equals zero!?
T	yes
S5	and we put the seven down there.
T	ok, so now, we start from the beginning. Step one.
S5	so: how often does seven fit into seven, ya.
T	how often?
S5	fit one time.
T	aha, where do we write down the one?
S5	under, uhm up, up. And then: we already have the answer but that doesn't matter. Then, we do, then we: because seven times one equals seven. And we put the seven under the seven. Then we subtract, so seven minus seven equals zero.
T (13:05)	so: [writes 285 / 5]. Now, step number one.
S4	divide, division, seven, uh, five, uhm twenty-eight
T	divided by?
S4	twenty-eight
T	by?
S4	five
T	by five, ok. How often does it fit in?
S4	eight times. Uh, no (…)
T	how often does it fit into the fifteen? Uh, in the twenty? Do you know that?
S4	it (.)
T	ok,we'll do it together: five, next one is? Ten,
S4	four!
T	four times, ok. Four times five would be twenty. Ya
S4	five.!

T	so five times, it's nearer, ya!? So: it fits five times into the twenty-eight. Where do we write down the five?
S4	up there!
T	up on above, ok, so, second step:
S4	we divide by (times) five
T	so we multiply now, ya!?
S4	ya
T	because five times five is?
S4	twenty-five
T	twenty-five. Where do we write the twenty-five?
S4	uh, down there
T	ok, below. So, third step is?
S4	five times twenty-five
T	no, third step is, you have it in your book.
S4	line
T	line. And then we calculate the minus. Twenty-eight minus twenty-five. How many does the twenty-eight have more than the twenty-five?
S4	uh, three?
T	aha, what do we do now?
S4	now, we-
T	here, look:
S4	ah ja. We put the five down to the three.
T	to the three. Good, now, what do we do?
S4	write it, five, five,
T	thirty-five divided
S4	by five is five, no, thirty-five.
T	no, how often does it fit in?

S4	eight-
(…)	
T	ok, so how often does the five fit into thirty-five. So we know, look: in the twenty-five-
S4	seven!
T	So: where do we write down the seven?
S4	up there
T	above. And what do we do then? Second step? Because-?
S4	because (.) uh thirty-, uh, five, e
T	because five times
S4	five times thirty-, thirty-, seven equals thirty-five.
T	thirty-five. Ok what do we do now? Step three.
S4	draw a line and the answer is zero
T	aha. And what is the answer? How often does the six fit-
S4	fifty-seven
T	fifty-seven times.

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T (03:46) and now, first i want to know: what is a square number?

S1 it is a number, like if you take for example two times two is four, then four is a square number.

T yes, so what happens with the number? What did you do with the number? If you would explain it in a sentence. So what did you do with the two?

S1 uhm you times it by itself!?

T good, ya, so: a number, or any number times, or multiplied by itself, no, eh, if you do this, so if you take a number, multiply it, then you have a square number. So the outcome is the square number.

T (11:33) (…) and when the answer is a squared number, then you shade it.

Ok, now, why are they called squared numbers?

S2 because you can square (around it)

T yes, if would take stones and we would want to show it, then it would look like a square
(…)
And: another thing is why they are called square!? What is the short form? How can I write the short form of three times three? I can write it even shorter. Can you remember?

S3 (?)

T yes, so in this case, it would be a three and a small tiny two up there. Can you remember? That was the exponents, so it was three squared.
So, what is the first square number?

S4 one!

T ya, so we write it down like this: one times one is one squared so it is equal to one

T (22:40) So, what is a prime number? First of all: what's the definition!?

S5 prime numbers are odd, we can't divide them by two and they just have two factors: one and itself.

T correct, so prime numbers have two – only two – factors, one and itself. Now, to the question: are all prime numbers odd? Is that correct?

S6 mostly

T mostly!?

S6 except two!

T what do we know about the two?

S5 the two is the only prime number that is even.

T and? What do we know as well? What's the second thing?

S7 it's dividable by two

T it's divisible. And yes, and one. (…) it is the only one that is even, and it is?

S8 it's the first one!

T it's the first one. Ya!? Not one.
So, the smallest prime number

S9 (25:12) but I think eleven is no a prime number?

T why not? Well, we can go through it:

(…)

T (31:25) now, my next question is: if we look at the five, how can I recognize multiples of five?

S8 it always stops with a zero or a five

T yes, so, can the fifteen be a prime?

Stds ya, ya? Ya! No.

T now, again, a prime number has two factors the one and itself. Now, A. said that all numbers that end with a five or a zero is definitely a multiple of five. So these numbers, the five fits in. so can the fifteen be a prime?

S9 oh, I know! No, no

Stds no, no

T no, because the five fits in, so that's already a third factor

S9 so we cross out!

T so what can we cross out?

S10 all fives

S11 fifteen

T only the fifteen?

Stds all fives

S9 the whole five column

T the whole five; because all the numbers here, ending with a five, they are multiples of five.

S12 and the zero!

T and the zero. So here, this column is already crossed out
(…) so, we crossed out all two-numbers and all multiples of five.

S10 and the rest is now?

T what's with the seven? Is it prime or not?

Stds ya, ya, ya

T yes, it is. And what's with all multiples of seven?

S11 ya-?, no! no!

T so what are the multiples of seven?

S11 seventeen, twenty-seven

T multiples! No, no

S11 oh, sorry, eh, three and four!

T huh? Multiples!

S11 so times!?

T yes! So, multiples of seven.

S13 fourteen!?

T fourteen. Is already crossed out. Next one!

S13 twenty-one!?

T oh!

S13 twenty-eight

T so, wait a minute, twenty-one we have to cross out because it's a multiple of

S13	twenty-eight is already
T	ok. Next one?
S13	uh thrity-
S14	-five

T (02:20)	now: what is a sequence?
S1	mmh, the amount of something?
T	no. if you look at a sequence, how does a sequence look like? (.) for example, this is a sequence (…) how would you try to explain it
S2	(one number line) which is always like in the same, like this (?)
T	same pattern, or the same amount, ya!?
S2	yes.
S3	I would say it's always two plus two plus two
T	but is it in every sequence like that?
S3	no!
T	ok, then try it without plus two
S3	uh, another one or this one?
T	no, you have a row of numbers and it increases, for example, by the same amount, or decreases (…) and then we write down: so, when numbers, or pictures follow a rule or a pattern.
T (07:35)	now, the new thing is, if we look at this sequence, there are numbers, in a certain order. And each number, so this is a value, so the first number in the sequence has the value two, the second number in the sequence has the value four, and so on and so on. Now: each number in the sequence, so this one, this one, this one and this one, we call them terms. So each part of a sequence is a term. So this is a term, this is a term, this is a term and this one as well, and it goes on and on and on. Because it's actually normally never ending.

(09:00)	(…) and now, of course, each term has a position. So, this is, has a name, we give it a name, that we know which number has which position. So: this here has the first position, coz' it's the first term in the sequence. So this means, this is term number one. This is term two because it is the second value in our sequence. This is the third one, the fourth, and it goes on and on and on. And if we do not know which position it has, because we don't know if it's the eleventh or the hundredth, then we say it is term n. because it's a number, that's why we give it the letter n.
S4	Why n?
T	because of number.
S5	can it also be k?
T	yes, it could be, but we call it n.
T (12:30)	So, what would you write down here? When they ask what is the pattern of the sequence? Or what is the rule of the sequence? (…) What happens in the sequence? What do you always-? What do you have to do to find the next sequence uh, term? What do you have to do? Look at it: two, four, six, eight, and the next number would be?
S6	ten?
T	and what did you do?
S6	plus two?
T	ah! And not plus: we want to have the nicer word, is?
S6	add
T	add. So the rule in this case is- when they ask for the rule is, what do you have to do to get the next number in the sequence!? Then, we would write down: the rule is: add two.
T (15:50)	if you look at the key words (…) there are three more (…) son one is consecutive. (…) what does consecutive mean?
S7	(it's like five plus six)
T	what are you thinking about? You're on the right track but: what are you thinking about? Five plus six is?
S7	eleven?
T	ya, and what kind of number is eleven?

S7	odd?
T	aha, and what did we talk about, by odd numbers? When we have even numbers and we divide them into two equal parts, ya!? Can you remember? If we divide a odd number, then we do not get two equal parts. How do the numbers look like? They are?
S6	equal not
T	(…) ya, they're not equal. But what, how do we call the name, uh the numbers? They are?
S8	unequal
S9	consecutive?
T	yes. Because five and six are consecutive numbers. Because the six follows the five right away. So it is that these are consecutive numbers here. They follow each other. So it's the same like five plus six, or five and six. Six is consecutive because it follows. It's the term or the numbers right next to each other. So if I would say right down the next, so I would start with this one, with number seven, and I would say: write down the next three consecutive odd numbers, so write down the next following odd numbers, and that would be then?
S10	eight
T	is that odd?
S10	oh
S11	nine
T	nine.
S11	eleven and thirteen.
T (18:45)	then, we have finite. What's that? F-i-n-i-t-e
S12	I just know what infinite is
T	what is infinite?
S12	uh, that it never ends.
T	it never ends, so what is finite?
S12	it ends!?

T	ya. (…) infinity never ending. So that's a never ending sequence. And finite is then of course ending. It stops then at one point.
T (19:55)	so it is six, twelve, eighteen. So what follows then?
S6	twenty-four? Thirty, and thirty six.
T	then we have five, eleven, seventeen, twenty-three, A. the next three ones?
S3	the next three ones: twenty-nine, thirty-five, forty-one.
T (23:35)	we have one, five, nine, so and we have to, in a sequence, when you have a sequence, that what you add, subtract, or what you have to do is always the same. It never changes. It stays always the same. So if we look at this here: from one up to five, it is plus four. So what do we have to do when we have two hops?
S3	three!
T	here, so we have to add then? Two hops, then we always add what?
S3	Always add two.
T	yes,
S3	and then it's seven and then it's eleven!
(…)	
T (25:50)	so here you could see you have to subtract ten, so if you need two hops, you just half it. And then it's always minus five.

T (02:20)	one, two, four, eight, what would we write down?
S1	times it by two
T	ya, times it by two. What is the other word by times it by two`
S1	multiply
T	yes, multiply by two, or? What do we say as well? A special word – only times two!? We have a special word for it. (.) when you would see me twice, then you see what?
S2	double!?
T	yeah, double.

T (03:00)	one, two, four, seven.
S3	add every time one, then add two, then add three, then add four, then add five, and so on.
T	ok, so we always add one more than before. And then we have one, two, five, ten and seventeen? What is- what would you write down?
(…)	
	Look, when we write it down, it is one, two, five, ten, seventeen. Look at the hops!
S4	add the next (? numbers)?
T	so we have here: how many do you add?
S4	you add one.
T	then, you add?
S4	three.
T	here?
S4	five.
T	so, add the next odd numbers.
T (09:33)	when they say: generate a sequence! What do you have to do?
S4	think of numbers and put them in line!?
T	so what did you do, actually, other word now for generate.
S4	(think)
T	think or create. So you do something when they say generate, that you do something. (…) When we have a sequence, what is a rule, then? (.) like, for example, I have the sequence two, four, six, eight, what is the rule?
S5	add two!?
T	yes, so this is the rule, that you write down, in words, what you have to do to generate the sequence. And then we have continue. What does that word mean?
S6	continue means you go on.

T	yes, and then the last word is unknown!? What is unknown? Other word? How would you describe unknown!?
S7	(missing?)
T	?ya? it's a number in the sequence we do not know.
T (11:30)	so, the new word is the term-to-term rule. (…) is what you have to do to come to the next term, to find out the next value. (…) to find out the next term from the previous. What is previous? What does the word previous mean?
S8	that was already!?
T	ya, the one before.

T (00:35)	the second term is six. Each term is double the previous term. They want to have the first five terms: one, two, three, four, five. First of all, five numbers. Now the information is: the second term is six. What is the second? Where do I write down the six?
S1	you write it down by four a and then you skip one and then you write it down
T	yes, six. This is the first term, this is the second term. This is the third term. First of all that. Then they say, each term is double the previous term. So what do we do in the sequence?
S1	so, you want me to figure out number one first?
T	no, first of all, what do we have to do, the sequence's rule is?
S1	uh, double the previous number, so for six, you have to double it. That would be twelve.
T	ya, so we double it because it is times two. Twelve
S1	then twenty-four, and forty-eight.
T	so, how do we figure out the first one?
S1	we just divide by two.
T	yes, the opposite.
S1	the opposite (?) so it's three.

T (03:38) so if we look at it, the first term is twelve, it's correct. Each term is half the previous term: almost. Almost. The last one is wrong. Oh, that's tricky.
So, if we would draw it, ya, if you're not sure, because dividing fractions is tricky, ya!?
So if we look at it: we have one and a half. We have one. And we have a half. Now what do we do? We divide them into? Into? What did you always do?

S2 halved it!?

T you halved it. So, what do we do? We halve it!? Half of one is?

S1 one half.

T how many?

S2 a half!?

T a half. And now, we halve the half. And what is that?

S2 a quarter?

T a quarter. How mean. If we now put these two parts together, how many do we have? How does the fraction look like?

S2 three quarters?

T three quarters! So it's not minus three, it is three quarters.

T (08:15) we'll look at it. So we have: the first two terms are two and three, that's easy. So the first two terms is: two, and three. The difference between two consecutive terms is one greater than the difference between the previous consecutive terms.
Ok, what is previous? Uhm, sorry, consecutive? Which case? In this case, which numbers are consecutive? Here in this sequence?

S3 two and three!?

T see! Two and three! Because they come after each other. So that's why they're consecutive. Ok, first of all that. Then, we read: the difference between two consecutive terms is: so, first of all, is one greater than the difference between the previous two consecutive terms. First of all: what is the difference between these two consecutive terms? What's the difference?

S1 it's one.

T it's one. So, first of all that. And now, they say: the following consecutive terms, the difference is always one greater than before. So now, we have ere: the difference is one, so what is the difference for the next two consecutive terms?

S2	(one again?)
T	no, because, again: they say: here is: the difference is one. And now, they say, each following consecutive, the difference between the following consecutive numbers will be always one more. Than before. So: here is one, so the next one is? The difference will be?
S2	two?
T	two! And here, the difference will be?
Stds	three
T	three. Because always one more. The next one will be?
S4	four
T	four. So then, what's now the sequence?
S5	uh, the sequence is always add one!?
T	ya, so we have two, three, the next one is?
S5	four. Uh five!
T	five. Then?
S5	seven.
T	I don't think so.
Stds	eight
S5	uh, eight.
T	eight. And then we have?
S5	oh, wait, plus four, it's twelve!
T	twelve. Ok. That was a tricky one.
S5	they didn't explain it very well. (…) Why didn't they just say: always add one more than you added before that?
T (12:45)	it says describe the sequence in words in another way. So that is the rule! So they want you now that you write in words what to do. So we'll do the first one

	together. Is? Each term is one more than the five times table. So: how will the sequence look like?
S3	uh, wait I have to check
S6	one more than the four times or the five times table?
T	the four times.
S3	ok, then five, nine, thirteen, seventeen, twenty-one.
T	yes. Because of course: four plus one, eight plus one, twelve plus one, and so on. Then, what is the rule?
S3	four times table and add one.
T	ya, but they want you to write it in a other way than they wrote in the book. So, the easiest, is when you say: first term. With which term did you – with which value did you start?
S3	with four?
T	no
S6	with five!
T	so, the first term is five and the rule is always? What did you always do then?
S3	uh, always, plus four?
T	and plus we don't say, we say?
S3	add
T	ok, so the rule is then: add four. So the easiest way is, you say: the first term is? Blabliblu, and you always have to do: blabliblu.

T (19:25)	ok, the second term is seventeen. Where do you write down the seventeen? Ok, then you write the seventeen down!? Now, we have: each term is three smaller than the previous term. So what do you have to do?
S1	(we add three to reach one)
T	ya
S1	so it's three less
	(?)

T	(…) subtract three each time. Perfect! Ya, subtract three. Because it is smaller. The magic word is smaller. So it gets smaller each time.
T (21:45)	for example, either they show you the sequence and you have to find out the rule, or they give you a sentence where you have to write down the sequence, and now: it's the other way, displaying sequences as, is as a picture. And now, when we look at the picture in the example, so, we have a picture at the beginning and then you see the sequence, that the matches (…) you see that the matches increase. So that's why you should look at the pattern because here, they ask you: find the rule for this pattern. So first of all, when you look at the picture, with which amount of matches do you start? (…) you can count them: easy.
S7	four
T	ok, so here, we have four matches. Now, if you look at the second picture, how many matches do you count?
S7	seven.
T	seven. So in the second picture, we have seven matches. In the third picture, how many matches do we have then?
S7	ten.
T	ten! And so on and so on. So we see that the amount increases always by the same number of matches. So what is the rule?
S7	add three!?
T	yes. So again, first term is four and add three. That's what they would want you to recognize. When they say: what is the pattern, then they want you to write down, always write down the first term, we start with this number, and: this and this happens in the next steps. (…) In this case, we look at a picture now, you have to see the pattern, so first of all: look: how many do I have in the first term, then you can write down: first term: so and so many. What happens!? Ya, write down the pattern that you write down the sequence. And then you see, by how many it increases or decreases or gets multiplied or divided- you write down the rule in words, ya, and then you can complete the following sequences.

T (01:40)	Now, the question is: would now be fifty-six in this sequence? Yes or no? because if you look at the sequence, actually, you can already recognize it or not.
S1	yes.

T	why?
S1	because there is the six, the sixteen, the twenty-six, it's always with the six in the end.
T	or a?
S2	one
T	yes. (..) and would the ninety-three be then in the sequence?
S3	no?
T	no. why not?
S3	because it's not with six or one?
T (03:10)	(…) and would sixty-three be in this sequence?
S4	uhm, no.
T	why not?
S4	(coz) it's odd.
T	it's odd! Ya. Because you can see when we always add to an even number another even number then the answers are always even, so that's why we cannot have an odd number.
T (10:45)	when we have a number (one circle), then we have three squares around it. So, if we have two circles, how many squares do we have?
S5	six?
T	six. Ok, when we have three circles, how many squares?
S6	nine
T	nine! (…) Ya: how many circles will there be in a pattern with fifteen squares? Now, if you always have to look at the ratio between the circles and the squares, look! So what do we just have to do?
S7	five circles?
T	yes, why?

S7	(it's always three more than before)
T	three more? When you have five, it's three- three more is plus three.
Stds	ya!
T	na na na: look: you have one and three,
Stds	ya!
T	you have two and six
Stds	ya!
T	so six is how many times two?
S8	three
T	three times. That's what it is. Coz' you said three more. Three more would be add.
S9	but it is also add, because, look: three plus three is six plus three is nine-
T	ah! Here add three, no, ok, sorry my fault.
Stds	ya!
T	will there ever be a pattern with thirty-two squares? Yes or no?
Stds	uh uh
T	why not?
S7	because there is only thirty-three in the three times table
Stds	yes
T	very good.

T	eighty-three?
S10	no: sixty!
S11	sixty-one
T (14:35)	we'll do it together. So row six would be?

S10	twenty-five
T	twenty five. Then?
S10	twenty-nine.
T	then?
S10	twenty-three, oh! Thirty-three.
T	next one!
S11	it's sixty one then
S10	ya, see!
S12	no, it's fifteen rows
S11	ya!
T	here, let us do it together now! We had (sixteen) was how many? So I. said four more, so he said twenty-five. Then the seventh-
Stds	yes, ya
S13	ya, eighty-three.
S10	sixty-one!
T	so: how many now? (?) ok, eighty-three, we'll see. It's correct.
T (15:35)	so, how many rows of four would there be in a pattern with one hundred-one dots. When you think that fifteen is eighty-three (.) what do you think? So first of all what is the strategy – how did we find out this eighty-three? (?) (…) if you look at the patter, did you see some sort of a rule? Or some sort of a pattern?
S14	yes
T	because look, the first row, they displayed it already, has one row of four and one on top, see it? Now, the second row has two rows of four and one on top. See it? The third row has three rows of four and one on top. Now, think a little bit back to the primary section where we had these, where we used these pebbles for multiplication facts. Look: one row of four. Which multiplication fact is it? One row is four is which multiplication fact?
S15	four?
T	one times four. So: what is one times four?
Stds	four!

T	and then the one on top
S15	five!?
T	five. So, again, two rows of four is how many?
S15	eight?
T	eight. One on top?
Stds	nine.
T	aha. Now, third row. Three rows of four are how many?
Stds	twelve
T	twelve, one on top
Stds	thirteen
T	so what do we always have to do, actually? (.) what is the rule or what is the pattern that we have?
S16	add five? Uh four?
S4	(four and one is) eighty-three is, it's not eighty-three. It's sixty one!
S17	ya!
T	it's sixty-one!? Why?
S4	because if you have fifteen rows of four, It becomes fifteen times four, and then one on top and that's sixty-one.
(…)	
T	again, look, they say it two rows and then they say: add four. Then, go back to the primaries, think of the pictures we drew the whole time, and these were the multiplication facts. The same is here: four rows of four. So this box here is sixteen. And one on top. So that means when you have fifteen rows, then each row has four pebbles, so that's why we calculate fifteen times four. And fifteen times four is? Sixty. And there is always one on top. So sixty, one on top is? Sixty-one. So now, think the picture: you have hundred and one dots. You always have rows of four and one on top. So how many rows? Now you have to think the other way around. How does the picture look like?
S16	mmh, you have forty-rows?
T	forty rows!? So if we would have – we'll check it! If we would have forty rows then we would have forty of four pebbles. So four times forty is?

S6 hundred fifty?

T so too many. So: you have to think, how many rows of four and one on top is one hundred and one? Mmh, multiplication fact. (.) so, which number do I have to multiply with four and add one and I get one hundred and one?

S17 wait! Uhm (.)

T one of the most important times table facts you have to know. How many? (.) so, it would look like this: if I would write it down, it would be something like huh times huh plus one equals one hundred and one

(…)

S15 tweny-five?

T of course! Because four times twenty-five is always hundred. One on top is one hundred and one.
(…)
Can there ever be an even number of dots in a pattern? If so, how can you prove it? So, will there ever be one with an even number?

S6 no!

T because?

S6 because, there is add always one.

T one?

S6 one dot

T one dot more. And always when we would leave the dot away, then the answers, the numbers would always be?

S16 even!

T even. Because it's the times table of four. And in the times table of four we always have even numbers.

Second file:

T (00:00) input, something happens, and then we have the output.
(…) so it's always when you have a value and you change it into something else. For example, we have Celsius and we want to change it into Fahrenheit, that we throw the Celsius into the function machine, in the machine, it turns the value into Fahrenheit and spits it out.
(…)
Input value is the value you put into the machine
The function is what? The machine performs. Ya?

And output, of course, (…) is the result that the machine puts out.
These are the important terms you have to know. (…)

So, for example, I can do more then only multiplying for example. So, I can also attach to my machine I have here, another machine, and another machine and another machine. Every time when the first machine spits out the value, it goes into the next machine and something happens in there and then it goes into the next machine and something happens in there and then it gets spit out. For example, I start with the – for example, I can start with the three. And in my first function machine, I multiply it by three, for example. So the three goes into the first function machine, multiply it by three, so it turns into a nine, this value is now, goes in, right to the next machine. It goes further into the next machine where we add four, ya, so it's multiplied by three, it turns into a nine, it goes into the next machine where we put four on top, so we'll have thirteen already. Now, the thirteen goes to the next machine where we subtract ten and now, at last, it is finished and it spits out the output and what is the output then?

S1 (?) so it's three.

T (07:10) ya. (…) so you can attach one machine to the other. That's possible.
The cool thing of function machines is that you can generate a lot of numbers. So, I can throw in any number into my function machine and of course the output will always be different. So, for example, I don't want to generate the three, I would now generate, for example, the seven. So in this case, I can also, if I want to, throw another number into my machine and the same thing would happen with the seven what happened with the three. So the seven is thrown in, it's multiplied by three, so I have twenty-one, then I add four, then it's twenty-five, then I take ten away, so it spits out? Fifteen. Or, for example, I want to throw in a four. So the same thing: it's multiplied by three, I have twelve, I add four, I have sixteen, I subtract ten, and then it spits out a six.

S2 what happens if you put all the three numbers, the seven and the four, inside the machine?

T no, because only one fits in. one after another.
(…)
So the important sentence is: a function machine is a rule, actually. Because it tells you what to do with the first number you have. (…)
When you input the counting numbers, so that is meant with one, two, three, four, five, it will generate a sequence. Ya, because, we always have the one, something happens with it in the sequence, then the two, something happens, if we look at the rows that we had, in this case, it is as well, so if I would start with the one, and it's timesed by four, I have a four, if I throw in a two then, I have a eight. And if I would write the result in one row then I would have a sequence.

S2 really? What is you (?) if you throw one sequence in there?

T I don't know what you mean.

S2 like if you throw in five, ten, fifteen, twenty?

T	no, the sequence, it is the counting numbers: one, two, three, four, five

T (00:15)	so first of all, what is clock-wise? What does this term mean?
S1	uh, like the clock turns.
T	ya, (…) now, if we look at a circle, because we can see this is a circle, when we have a full turn, how many degrees is it? A full turn.
S2	(?)
T	three hundred sixty. So, a full turn is one circle, so I turn one full turn around. This is a full turn. A half turn is how many degrees?
S3	hundred eighty
T	a hundred eighty. So it's a half turn, so a half a circle. And a quarter turn
Stds	ninety
T	is ninety. Now, she has the shape. That looks like this here, and now she turns it quarter turn clock-wise. So how does the shape then look like? Like this, look. (…) because it cannot be, some of you drew this shape flipped to the other side, that's a turn of one hundred and eighty. So you turn this shape ninety degrees. Only a quarter turn.

T (03:40)	how much does she pay altogether? So, what do we do? Because now you have to calculate in different steps. So first of all: how much does the cheddar cheese cost? What do I have to do?
S4	she buys two hundred grams of the cheddar cheese, so you'll have to double, uh, eighty-two, so it's then one hundred and sixty-four.
T	perfect, pence, ya!? So: one hundred gram was eighty-two, she buys two hundred grams, that's why she doubles it. Times two. now, (…) edam cheese: one hundred fifty. What do you have to do?
S5	you do for one hundred grams sixty-six pence and plus, and thirty-three because it's the half. And then it's ninety-nine. And then you add the two together
T	ya. Because- why? How do you know you have to add up all the answers together?
S5	because then, you have the final answer?

T yes. And they wanted to know how many togeth- how much money do we have to pay together. So, hundred grams of edam cheese was sixty-six pence. Fifty grams, that's half of the amount, so that's why you have to halve the price, if you add it up, then we have in total then of course one hundred fifty gram. And because they want to know how much money do I have to pay at the end, altogether, I have to add up one hundred sixty-four with ninety-nine. And how much money is it?

S6 two hundred sixty-three and (there) is a comma.

T ok, pence, and how many, when it would be pounds?

S6 ya: two pounds and, uhm

T sixty-three.

T (07:26) Sed buys some cottage cheese for one thirty-five. How many grams of cottage cheese does he get? How can you find it out?

S1 uhm, forty-five plus, forty-five is ninety, ninety plus forty-five is one thirty-five

T one hundred forty five, first of all

S1 ya, and then I, and then, coz' I used the forty-five three times, I have to times the one hundred grams times three is three hundred grams, so he gets three hundred grams

T perfect, so M. added the forty-five pence so long until she had the amount Sed pays. So it's one hundred thirty-five. This is what he pays. So that's why, she adds it up so often until she has the same amount. And then she counted: how often do I have forty-five pence, or how often did I add forty-five pence. Three times! And we know that forty-five pence is equivalent to one hundred grams. So if I pay three times forty-five grams, uh pence, then I buy per forty-five pence hundred grams. So when I do it three times, I get three hundred grams.

T (09:27) now this thingy here, this a thing, of course you have to know what a perimeter is. What is that? How do I find the perimeter?

S1 it's the outer side

T ya, so I walk one way around the shape. So what do you just have to do to find the longer one?

S1 uh, you have to count the sides.

T yes (…) so that's what you did. The most of you did that. You counted the sides and then you said the one that has the most sides is the longest
Now, what is the line of symmetry?

S5	when you have it, it looks completely (the same?)
T	so, as if you would put a mirror in the middle and when it reflects, it has the same shape on the other side.
T (11:30)	so, one hundred forty-four divided by six. How do I calculate it?
S7	hundred divided by six and forty-f, uh, wait, like, can I say just the answer, or?
T	uh uh, the method. You have a method, don't you?
S7	im doing it like, but i
T	ya, that's perfect, what did you do? Why did you take the twelve?
S7	because six times two is twelve
T	aha, so, it is: one divided by six – does that work out?
S7	doesn't work
T	no. so that's why we take the fourteen. So how often does the six fit into the fourteen?
S7	two times
T	two times.
S7	that will be twelve.
T	twelve
S7	and then you like subtract them
S8	to bring the four down and then you have the answer
S7	twenty-four. And then you divide twenty-four by six and it's four. And you write down twenty-four again and then you write zero. And then the answer will be twenty-four.
(…)	
S9 (13:15)	so, one hundred twenty divided by six is twenty; twenty-four divided by six, because the twenty-four is left-over, is four, forty-four, ya! And then twenty plus forty-four is twenty-
T	no: four. (…) four times. It fits four times into it.
S8	right. I will cross that out. I didn't see it. Is twenty-four. And then you add it up.

(…)	
S10	so: twenty times six is hundred and twenty, twenty-one times six: and so on and so on until you hit the- uh,
T	oh god, that takes too long.
S10	no: I started with twenty times.
T	so you just add until you get it.
S10	ya!
T	but what happens if you have something like seven hundred forty-four
S10	bad luck, no, then you just start by (.) one, (…
T (15:55)	now, quarter to eight. Which time is quarter to eight?
S5	it is like fifteen minutes to eight!?
T	ya, how would you write it down in a digital time?
S5	eight fifteen, uh forty-five?
T	are you sure? Quarter to eight
S5	no, fifteen, no! forty-five!
T	ya, forty-five is correct. But which hour?
S11	seven
T	seven. Because it's to eight, we're not at eight yet. Six to eight!
S12	six, uh, (…) it's a seven, and then it has the little two dots thing, and then it's uhm fifty-four.
T	fifty-four, ok, because it's again to eight. It's not eight yet. Now: half past seven
S13	it's seven thirty
T	it's seven thirty. Because it is half an hour past
S11	past. Not before.
T (18:40)	so, this is the height of the mum and half of it is the boy. He is half as tall. So, freddy is one meter shorter than his father. Freddy's father is one hundred

	eighty centimetres tall. So his father is here and is one meter taller than his son. So that means, what do we know now?
S14	if you subtract a hundred from hundred eighty then it will be the height from freddy.
T	yes, so look: so this is the height of the father. This is one hundred eighty. And this here, this height: the difference between the father and the son is one meter, so one hundred centimetres. So how high or how tall is now freddy?
H12	freddy is eighty
T	eighty. So, because here is the difference one meter, that's why you subtract it. That was totally ok. That's why he is eighty centimetres tall. (…) that freddy is half as tall as his mother. So, other way around, what is his mother?
S1	uh, we have to times the eighty times two. is one hundred sixty.
T	of course. Because if freddy is half as tall, then his mother is double
S11	as tall as him
T	ya.
T (23:30)	before you can even compare these capacities, you first have to bring them in one unit. So, the easiest would be to convert one point seven litres into millilitres. How many millilitres? What is one point seven litres in millilitres?
S14	one hundred seventy millilitres? Uh thousand seven hundred millilitres.
T	yes. Thousand seven hundred millilitres. How many millilitres fit into a litre?
S5	one thousand?
T	one thousand. So that's why. When the one is a thousand, then the seven has to be hundred. Otherwise it doesn't work out. So, how many millilitres then is one point zero seven liters?
S5	one thousand seventy?
T	ys! Because you have, here is the thousand, you have zero hundreds, and that has to be a ten! And one one quarter litres. How many millilitres?
S6	ya, ok, that's one, one two fifty
T	I would like to have the full number
S6	one thousand two hundred and fifty millilitres

T	correct. Now, as soon as you have all capacities in one, in the same unit, you can compare it, ya!? And then the correct answer would then be?
T (26:13)	on two cubes, we have crosses and on three cubes we have circles. Now, they stick them together. Now, they ask: if you would look at this 3-D-thingy, you would have this in your hand, how many crosses would you see? And the most of you wrote down twelve. Because you stopped to think immediately as you saw that: cubes, six sides, two: twelve. But: to think a little bit longer, it is of course: do we see all sides when they are attached to each other?
S1	no.
T	no! so how many crosses do you see?
S1	eight.
T	eight. This is how many you would count. Now, the same with circles: how many circles do you see? And you only see the ones that are outside.
S13	fourteen
T (27:55)	mina has five more marbles than kirsty. Kirsty has two more marbles than seb. Altogether have thirty marbles. Now: how can I find it out? (…)
S15	can I show on the board? (…) first, I drew the (towers?)
T	mmh, or bars, you drew, right?
S15	(…) (?) mina has five more marbles than kirsty, then here is five and seb has like two less than her, so this is two. and then I add the left over and together that's five plus two is seven (and two) is nine. So, and then I subtract from thirty the nine. And then it's twenty-one, so for now, we just have this part (?) so I divide twenty-one by three, and then it's seven. So: here is seven, and you have two more for her, so you add two plus seven, is nine, so she has nine. (And she had seven) more, so you add seven to it
T	(…) that you see the differences between the children. And the good thing, what she did is, that she put the bars in a way that at the end, when you take the differences away, that at the end, she had three bars with the same height. And that's why she just have to divide it by three. And then she knew, ok, up to here, they are the same, they are seven. This one has two more, so that's why it's nine. This one then has five more, and that's why it's fourteen.
T (32:20)	you just had the possibility to guess. So what I say is that suet is twenty percent because if you look at this picture, then you can see that millet, here, would approximately fit two times into suet and three times into sunflower and into dry bred four, that's what you can see. When you add then the total of all parts,

	then you would have ten. And that's why twenty percent is not bad. Because for suet, has two parts of millet and that's why it's twenty. Get it? It's just a round about. Just approximate. Because they say estimate. And you can only – ya, that's what they would want you to see is that millet about, it's two times in suet, three times in sunflowers and four times in bred crumbs. (…) Again: coz millet is ten percent and you need three parts of millet for one sunflower, or for the sunflower seeds, that's why three hundred.
S5	if you estimate it like that.
T	ya, this is the only possibility
S5	but, if you estimate it, you measure it with a ruler, and in degrees
T	you cant. That's not estimating.
S5	oh, but it's still drawn if you do it, estim-
T	ya, but that's not what they are asking for. They don't want to have a accurate answer, they said estimate it. And that's why they just want to see you that millet fits two times into suet. That's the whole thing. Because otherwise they would say measure.
S5	ya.

T (07:50)	work out the output for this function: one, two, three, times four, plus one, ya!? And then b: reverse the order of the functions: add first one and then multiply it by four. (…) So, ya, are they the same as a?
S1	no!?
T	no, ya!? So, if we change the order, then the answer is different. So that's why when they give you a order, the way you have to calculate it, then keep the order. Coz as you can see, if you are sloppy, then it would, we would have the wrong answer. So take it serious.
T (10:50)	actually the same that we had, functions, that we have the input, we throw the numbers into the machine, something happens with the number, and then it gets, well then the machine spits it out and that's our output. So this is how we can imagine it. And this was easy because they already told you the input and you just had to do that what was standing on the machine. So now, we have the heading is finding the function. So, last times, we always had an input, we did something and then we looked what is the outcome. Now, in this case is the input is missing, we know we

	have to do something with the input, so this input would be, for example, be multiplied by five, and the output is ten. So, what do we have to do?
S1	it was (?) two
T	how did you know that it is? What did you do?
S1	two- two times five is ten
T	ok, but if you, what can you do with the ten? To get the two?
S2	you can divide it by five!?
T	yes. So you do the reverse operation. So if we would look at this, if we would have, it would look like that when we would draw it. Then it would be that we have a number (…) n, ya, for number, so n we are looking for. We times this number by five and we get ten. And to find out the number, we just do the opposite and the opposite of multiplication is?
S3	division!
T	ya, so that's why we divide the ten by five and get two. so here it's a multiplication and here divide.
T (14:50)	so now they ask, well, when I have the input five, or seven, or ten, or thirteen; I throw it in the function machine and it is spit out, so the five spit out as a fifteen, the seven is spit out as a twenty-one, the ten is spit out as a thirty, and the thirteen is spit out as a thirty-nine. What happened in this function machine?
S4	times three
T	times three.

Class level: Grade 7

May 2013

T (09:20)	So, when we want to find out the lowest common multiple or the highest common factor, we always deal with at least two numbers. So for example: sixteen and thirty-six. (…) What do we have to do? What's the first step?
S1	Put them into their prime factors?
T	yes, so we put them into their prime factors. (…) Which one is the smallest prime factor? one?
S2	two.
T	two! Yes! So, the two is the smallest prime factor! (…) NOT the one! (…)sixteen: How do we start?
S1	two and eight. Or four and four.
T	ok, two and eight – then we split up the eight in?
S1	two and four
T	yeah
S1	and then four into two and two.
T	two and two. Ok.
S1 (11:05)	So you got four twos
T	so we have the eight – the sixteen has two times two times two times two.
(…)	ok, then we have the thirty-six. (…) We start with the –
S2	two?
T	With two. Two times what?
S3	eighteen?
T	good and then?
Stds	(mumbling) (six, nine) eh six and three
T	ah!
S3	and then – two and three
T	ok. (…) then we have the two, the three, the two, and another two

	(…) a typical mistake of a few of you would be this here. Is this correct?
S2	no
T	no? why?
S4	Because you can still break down the nine in three and three.
T	yes, don't forget it: the nine is a square number not a prime number. So you have to split it up
	(…) How should you now write the factors down? The prime factors.?
S5	ehm, when there are like, sixteen has four twos, and thirty-six has two twos and two threes, so…
T	How should I write it down? Like this?
S5	no, ehm, like, at the end of the twos from the sixteen, we write, like, two twos from thirty-six, and then you write the two threes.
T	yes, that's it. Very good.
S2	Why couldn't you also put the two twos from the thirty-six, ehm, into the front?
T	Yes (you can, …, you have to make) sure that the same factors are written under each other. (…) You have to have some sort of order how you write it down. (…) Highest Common Factor. What do we have to do?
S1	See the numbers which are the same and circle them!?
T	Ok, so in this case, what do they have in common?
S1	They're the same, like two-
T	So, what do they have in common?
S1 (14:20)	(all) the twos
T	The two!
S1	(the answer of this one)
T	And? Is this all?
S1	The answer is the same.
T	No – do they only have these – this – two in common?

S1 No, and the-

T ok. Look, so: What the sixteen and the thirty-six have in common is a two and another two. Two times two. So if we write it down, two times two, what is the highest common factor?

S1 four.

T ok? So it was: you write the factors underneath each other and you circle in the vertical way the common factors. And then you can see in this case it's the two twos.
(…)
(16:16) Now we have the lowest common multiple. (…) now, how do I find that out?

S1 you always check like the sixteen and the thirty-six – now, the sixteen has more twos than the thirty-six,

T So which twos do we circle in?

S1 the sixteen's!?

T ya. So the sixteen has more twos. That's why we want to have the twos of the sixteen.

S1 and thirty-six has more threes so you circle the threes.

T ok!?

S6 (hundred forty four)

T a hundred forty-four?

T (26:00) What are the prime factors of twenty-one?

S7 three and seven.

T perfect. Three and seven. And the eighteen? What did you do there?

S7 ehm – shall I say the prime factors or- ?

T ya

S7 three and six!?

S8 six?

T ok.

S7 then, ehm, three and two

T	good. What did you do then? (.) what did we do there?
S7	achso- then I say eighteen is two times two times three. And twenty-one is three times seven.
T	three times seven. Ok. And, again, please make sure that you have the factors, the same factors underneath or under each other. Ok, now, we want to have the highest common factor. We start with the highest common factor (…) So what do you have to circle in?
S7	the three and the- the threes.
T	so, the threes are common factors. Do they have any other common factors?
S7	no.
T	no, ok, the highest common factor, we already have. (…) No, LCM? Maybe an arrow that goes to the right. (…) Who has the most? Which ones are they?
S9	the three
T	the three.? Which ones?
S9	the, three.
T	ah, so, again, the eighteen has the factor two and twenty-one doesn't have a factor two. Who has more twos?
S9	the eighteen?
T	ah! So, we want to have the twos from the eighteen. The eighteen has the factor three, two threes, and the twenty-one only has one three. Who has more?
S9	the eighteen!?
T	so we want to have both, ok!? Now, the eighteen has no seven and the twenty-one has a seven. Who has more?
S9	the twenty-one
T	So, we want to have the seven from the twenty-one. (.) and then you multiply it out! (…)
(29:20)	Olivia, how did you split up the forty?
S10	two times twenty?
T	ok,
S10	and then two times ten (..) and then five times (two)

T	finished?
S10	yeah
T	ok, now, the forty-eight. (…) ok, which number fits in? which prime number? Its an even number, so which prime number always fits in every even number?
S10	the two?
T	the two. Yeah? So you just have to, now, halve it. What's the half of forty-eight?
S10	ehm: twenty-four.
T	twenty-four, ok. So which prime number fits in the twenty-four now?
S10	the two!
T	The two.! Good, and what's the half of twelve-ehm- what is the half of twenty-four?
S10	twelve.
T	which prime number fits in the twelve?
S10	two times six
T	and this is six. And then?
S10	three-. Two times three!?
T	(…) Now, which factors do both numbers have in common?
S10	the two?
T	yeah, and how many times?
S10	three times.
T	ya. So you have a two here, these two are in common, and these two.
S11	so: three up to the power of two or two up to the power of three! Makes nine.
T	no.
S11	no, not, oops
T	think first! Two up to the power of three is-?
S11	oops, I made a mistake, I mean two up to the power of three is eight.

T eight, ya, so the highest common factor is eight. And now, we have the LCM!? So again, we want to have the most, it's the vertical – the – horizontal

S12 its- here, the- two times two times two times three times five!?

T so we want to have these twos, we want to have the three, and, of course, we want to have the five. (…) and the answer is?

S11 two hundred forty?

T two hundred forty - eight times (.) one hundred twenty.

S11 no- it's, ehm, two times two times two times two-

T ah, ja, it's ok, I –i- forgot one.
(…)
(34:20) What is a ratio?

S1 a ratio? If you have sixteen balls. And the ratio is two to one to three, I don't care, two to four to eight!

T now we have fourteen balls in total – but that doesn't matter.

S1 so, you have to first, add all up them. So this would be a fourteen.

T But what is the difference now between these – this part and this part and this part? If we would have fourteen balls?

S1 it's the amount of the whole thing that belongs into this group.

T And what is specific about the group?

S1 Let's say- two ehm- two, like, green, red, blue, they're two of- like…

T like that!?

S1 yeah.

T so in this case, for example, the typical story, we love it. We have fourteen balls. In there are green, red and blue balls and I had two green balls, four red balls and eight blue balls.
(…)
Now the first thing is: the ratio- can a ratio be simplified like fractions? Can I simplify ratios?

S13 yes!?

T ok, so if we would now look at Lukas' example: can I simplify two to four to eight? Yeah? And it would be?

S13 one to- (four)

T	ok, got it?
S14	I do it different
T	yes, you can
S14	I add them up and I divide them
T	no no, it is just, when you look at the proportions here, you have two groups, you have four, and you have eight – so, can you just simplify, that was just the question So, first of all, you can simplify ratios in the same ways as fractions. So, of course you have to look at the numbers, you have to see if they have a factor in common and then you simplify. Maybe a more practical example would be, for ratios, is that I bake a cake and I have, so in the English system we always bake with cups, ya?. So I bake a cake and I have three cups of flour. Ehm, make four cups of flour, two cups of sugar and that's enough. Eight cups of chocolate. So: what is the ratio?
S15	so four, two, eight
T	Now, I want to bake a cake and I only want half of the amount. Is this possible? What would you do?
F	You just, ehm, you would add them up?
T	I would add them up – I only wanna have half of the cake. What would you do with the ingredients?
S14	halve it!?
T	yes, so how many cups of flour would you use?
S15	two?
T	how many cups of sugar?
S15	one
T	how many cups of chocolate?
S15	four
T	four. So what you did is – you simplified because you divided each number into two. And the ratio still stays the same. Because if I bake a cake, of course, the ingredients have to be in the same ratio, otherwise it won't taste. Or it won't work out or it won't be a cake afterwards.

	And now, if I take this here and I want to bake four cakes – what do I have to do? How many cups do I need? I want to have four cakes!
S1	four times four which is sixteen,
T	so I would need sixteen cups of flour,
S1	two, eighteen, eight, I mean
T	eight cups of sugar
S1	and, eh, thirty-two.
T	thirty-two cups of chocolate. So, you can also do it the other way around: you can also extend it.
S16 (40:20)	One which I didn't understand was somehow – they only gave you the ratio – like for example now two to four to eight, and then somehow, of chemical c, there is thirty grams. How much is there of a and b!
S17	They didn't tell you how much solution you have
(…)	
T	so we know: we have one information what is the information?
S17	like ehm, the ratio
T	no, what do we have now? the ratio is here
S16	yeah: that we have thirty grams of chemical c
T	so we know: chemical c would be thirty grams. And now they ask you what is a and what is b. (.) so, what do we have to do?
S1	Find out what one part is?
T	ah, and how do I do that?
S1	divide thirty by seven!?
T	yes!
S16	then it's like, with the never ending three, so it's-
S18	doesn't matter
(…)	
T	and then, what do you have to do in the second step?
S1	this times two and then times three

T	times three?
S1	yes, because then you have to answer for two and then…
T	ah:ja! (…) What you do is: you type it into your calculator, you will have this long number, and then-, because this is now equivalent to one part. The thirty grams are seven parts. That's why divide the part into seven parts, to find out one. And how do we find out two parts? What do we have to do?
S12	times it by two
T	two. So what you do is- ok, again: with the calculator: thirty divided by seven, then you have the result and then you times it by two. What's the answer? (…)
S16	eight point six
T	ya. And now: What's when I have six parts?
S7	twenty-five point seven (…)
T (47:45)	So if we work with ratios, everybody, we have parts and they tell you how many parts you have from each item, whatever. And the easiest way is always: find out how big is one part. As soon as you know how big one part is, then everything else is easy. (…) So: same ratio, same sea flower. What is the weight of each chemical in a tub of plant food containing eighty grams of the mixture? What are the eighty grams? Is it one of these parts or what is it? (…)
S19	all?
T	all! Of course. The total. So altogether, it's eighty grams. So what do I know now? So I always deal with parts. I know the seeds a, b, c is two, six, seven. And now we know, altogether is eighty. so what do we know as well?
S19	(that a is two times and b six)
T	aha, and c is seven. so altogether? How many parts are there altogether then? (.) two, six, and seven-
S19	- fifteen
T	fifteen. So: eighty grams is the total. So that's why, if this is the total, I have to add up all the parts I have. So I have two as, I have six bs, I have seven cs, so when I add them up, I have fifteen parts in total. What should I do now? That is the most important thing with ratios. It is the information I need.
S13 (51:10)	You divide eighty by fifteen then you have one part and then you multiply that number by a ratio.

T	I know that eighty grams is equivalent to fifteen parts. So to find out one part, I have to divide these eighty grams into fifteen equal parts and that is in this case-?
S20	five point three three three three (…)
T	(…) now we want to find out how many as, how many bs, how many cs. So what do I have to do to find out a?
S5	you have to take five point three three period times two!
T	(…)now, what do we have to do to find out b?
S15	(this times six)
T	and why? Where do we have this six?
S15	because there are six parts.
T	yes, because b has six parts and that's why we have to multiply it. And what is five point three period three times six? (…)
S20	thrity-two? – no – mmh
T	thirty two point-?
S20	nothing
T	mmh? Thirty two.!
S20	yeah, but that doesn't make sense
T	why?
S21 (54:00)	because point three three three three three anything – never comes to a full number
	(…)
T	in a year eight tutor group, there are six boys for every five girls. This is already a ratio. (…) Now, when we look, we have six boys for every five girls. What is the ratio? Five to six – you knew it, ok so (…) there are eighteen boys in the tutor group, so how many girls are there in the tutor group? (.) and the ratio has to stay the same. So what happened to the group of six? Now we have eighteen, what happened?
S20	(ehm, it got more?)
T	yes. By how many?
S20	by three?

T	aha, by three (.) so what do we have to do with the girls? With the amount of girls?
S20	times it by three?
T	times it by three as well. And then we have how many girls?
S20	fifteen?
T	fifteen. (…)
(58:40)	What is now the two hundred ninety-seven? Is the-?
S20	cakes!?
T	all cakes together. So two hundred ninety-seven is the total. (…) How many parts are there? Is it equivalent to? (.) so we have a ratio of four to seven (…) and this is the total. How many parts is this equivalent to? How many parts? We have four cream cakes and seven plain cakes. So the total would be then?
S20	twenty-seven-
T	no, how many parts? (.) [drawing cakes on board] (…) So, if I would now put these parts together,
S20	ah, eleven!
T (01:00:00)	ah! So the total is then four plus seven is eleven parts in total. Now, they say- what do I have to do now? First thing is, what I know now is, ok, I have eleven parts, two hundred ninety-seven is the total, the first thing, what I first of all want to find out is-? How big is? (.) ONE part. I want to always know how big is one. This is the first thing what you know, as soon as you know: ah, this is the total, I know this and this, then, the next step is, really, find out one part. How do I find out one part?
S20	divide the total by eleven
T	yes, is twenty-seven. And now the question is, look, this is, twenty-seven is one part. And now they ask you: how many cream cakes are there in one shop? What do you have to do? (.) so, in one part are twenty-seven cakes and I have four parts
S20	achso: twenty-seven times four!?
T	yes.

Class level: Grade 8

May 2013

T (02:05)	What was substitution again? (.) What does that mean when I say: ‘Please substitute – oh, [incoming student], good morning, sit over there – Julina!?
S1	So, maybe, if this, eh, letter, that x and above, all the – eh, thingies - is written x equals (?)
T	So substitution means to replace something. So if we look at this task: exercise five: ‘evaluate the following for question one to twelve’ … Then they say a equals three and ehm c equals two and e equals five. And in task number one, they say: ‘three a minus two’. And when they say evaluate, then they want you to solve the task. So, what do we have to do Julina? Easy Peasy…
S1	So, we do nine minus three!?
T	Ok. Please, when you substitute the numbers, put the numbers in brackets, yeah!? (…) So, if you substitute it, please please please, always write it in brackets. Because later on, when in the further tasks, we also have negative numbers, that you make sure, Luisa, that you (…), that the negative numbers, that you put them in brackets, that you don’t forget it later on. Because if you don’t write it in brackets then it would look like this and then you would have a total different answer. Because three minus three minus two is minus two and you forget to multiply. Typical mistake, Nasar. Ok, so what do I have to do Julina?
S1	three times three, its nine, and then nine minus two
T	Is?
S1	seven?
T	Just wanted to make sure (?) (…) Understood? Yes or no?
Stds	Mmh
T	Ok, then I want, for example, Alva, to calculate number ten. And that is ten a plus c plus e. What do we have to do?
S2	(.) Ehm (.) ten times three
T	yeah
S2	Plus c
T	What is c?
S2	Eh, two

T	mmh
S2	Plus five
T	Plus five. The answer is?
S2	Mmh – wait - eh – eight
T	Heh?
S2	No, no, wait, wait, no, I had it (…) thirty-seven, yeah
T	thirty-seven, yes, ok! And again, you multiply here: ten times three
S2	Oh, here!
T	Ok, you have it, Alva? ok

T (07:00)	Just as a warm up: Marvin: two m minus three. How do you solve it? (.) ehm: Anne!
S2	Ehm (.) so (.) ehm (.) so then its two – so you substitute m with two
T	Do I put it in brackets or not?
S2	In brackets (.) and then minus three. So then its two times minus two which -
T	- is?
S2	Eh -
T	What is two times two?
S2	Ok, minus four
T	Ah, ok, minus four
S2	Minus three so then it's minus one, negative one
T	I would be careful: minus
S2	Oh, minus minus is plus, ehm, negative, ehm, what? Seven!?
T	Yes. Because you have a minus four credit and another minus three credit and that why you have more credits in the end. Ok, now a tricky one (…)

T (08:35)	Three h minus twelve. Marvin, your turn now!

S3	Ehm, three, in brackets three,
T	mhm
S3	Ehm minus twelve. Eh, then its six – no: nine, nee, doch: nine, eh – minus twelve – eh – minus three!?
T	Yeah … ok. Good. Everybody knows how this works out?
Stds	Yes, yeah, yes
T	Also understood?
Stds	Yeah yes
T	ok
T (09:34)	X is equal to minus three and y equals two. (…) x squared: What do we do? Marlini!
S3	Mmh you do – also – minus three (squared minus nine is three) so you square minus three and its minus nine
S4	Positive nine 'coz minus (times minus) is plus
S3	Ok, positive nine
T	Why? Because:
S3	Minus and minus (…) because (minus and minus) is positive?
T	Yes (.) so that's why it's nine. Because if we square a negative number, Tim, what is always the answer?
S4	positive
T	It is positive. If I cube a negative number, what's the answer?
S4	It's negative [because (…)]
T	[It's negative. Times negative] Ok, now: whit is when I have three x squared? Now we have two – we have an exponent and we have a multiplication. What comes first? (.) Marie!
S3	Maybe three times nine
T	Yes! Why? Because first we would substitute

S3 (11:00)	Because the ehm x squared is one thing on its own and it doesn't belong to the three anymore. (.) so it's three times x squared and not three
T	Yeah but why can't we – because now we know that x for example is now minus three. Why can't we just eh multiply this first and then square it? That's the question. Because I think that a few of you if we have this task, some would first multiply. Why isn't that allowed? Nicholas!
S5 (11:50)	Well, first of all, minus three squared is one part of the equation, so that – it can't be – I don't know how to explain it but, like, ehm, it can't be that. Eh, you times the, eh, eh, the three and minus three and after that you square it. That's not right, because, eh, minus three squared: they're connected. So you don't square- you, ehm, (…)
T	First of all, it belongs together. What have you learned when you have different operations in a task? What do you have to follow, Anne?
S2	Bidmas!?
T	Yes.
Stds	Right
T	So you always have to keep this (…) And of course the indices and the exponents are first. But, they are attached to each other because it is a multiplication fact, as well, so, yeah. And then, the answer would be, then, thirty-seven.
T (13:30)	So, what do we do first?
S6	The brackets? So two times minus three
T	Like that, yeah, squared (…)
T	So, the first step is always substitute. So you replace the x with minus three and what is y?
S6	Y is two squared
T	Ok, so this is two squared, ok, now, we keep this in mind: bedmas. So now, what is squared? (…) Is the x squared, is the two squared?
S6	The two?
T	Only the two?
S6	And the minus three

T	Yes, so, everything that is in this bracket has to be squared, if I only want that for example the x is squared, then I write it down like this. So in this case, the three is not squared because only the x is squared. If both parts have to be squared, then I put the whole term into a bracket. Yeah? So now, what do we do now?
S4	Can I say the answer?
T	No!
S6	Is it then minus six?
T	First of all minus six. Because first, the bracket, yeah? And at the beginning, please, for the ones who still have problems, write down step by step. (…) no, what do we do now?
S6	Ehm, thirty-six -
T	yaha
S6	Minus four
T	Mhm
S6	That's thirty-two
T	Thirty-two. Perfect. Very good. Why doesn't the sign change here? Because we have a minus as well. Why doesn't the sign change? Coz look: here we have a minus six and it is a thirty-six. And here, we have a two and a minus as well, why is it four? Why does the minus sign (.) Why does the minus sign stay the same? Why don't we change it?
S4	Because its not in the bracket
T	Because it doesn't belong to the two
S4	Yeah
T	Because it's a positive
T (16:50)	We have two x in brackets squared minus three y in brackets squared (…)
S7	Ehm, two, eh, brackets, two, and then like another bracket, minus three, and then minus three and in bracket, eh, two, then square
T	ok
S7	And then, minus, minus six square

T	Yeah
S7	(Minus twelve)
T	So minus six squared, and, here?
S7	and, six
T	Ok
S7	And then minus twelve – no! twelve
T	It's a squared! It's not a times two
S7	Oh, yeah: thirty six
T	Thirty six, ya. Ok.
S7	And thirty six.
T	Which sign?
S7	Minus!?

S8 (18:50)	Ehm, two, then brackets, minus three. And minus four, in brackets minus three. Plus one.
T	Ok
S8	So, first it's two times minus three which is minus six
T	Eh
S8	Ah, so first the brackets! So, it's also six then because it – oh no, it's six – oops, I mean nine, plus nine, and the other one is also nine. (..) so two times nine
T	Yeah, is?
S8	eighteen
T	Eighteen.
S8	Minus four
T	Do you want to write it down? What do we do now?
S8	No, we can - , Don't you do then four times nine because -

T	Why nine?
S8	Because minus, no no, minus four, no- (.) is it minus four times minus three?
T	Yes! Because, again, the signe left to the term always belongs to the term. So the minus belongs to the four here in this case.
S8	Ok, so it's twelve
T	Twelve. Which sign?
S8	Minus?
T	Well, what do you have? You always said, you have minus four times minus three
S8	Times?
T	What is minus times minus?
S8	Plus. Ah! Ok.
T	Ok
S8	Plus one.
T	Plus one.
S8	is thirty one?
T	Is thirty one.
T	So again, the term, the sign left to the term belongs to the term. So in this case, you have to multiply minus four with minus three and then you have positive twelve
T (21:35)	X squared plus three x plus five. So, here is x. What do we always do first? (.) They say substitute. So what should you do in the first step?
S1	Substitute?
T	Ah, very good. (…) so what do we do? What is x? come on!
S1	Eh, x is minus three.
T	Aha. What do we do with this x? we - ?
S1	Square?
T	Aha, good, then we have a three. What is this x?

S1 Minus three.

T Minus three. And we add five. Ok, what do we do here?

S1 We square!?

T Yes! So, mmh, minus three squared is?

S1 Nine

T Ya. What do we do then?

S1 Well, we write it down!?
(…)

T What do we do here? Do we leave it like that?

S1 No
(…)

T What do I have here? In between the three and the minus three? That's why I put a bracket around it. That I always keep in mind. What do I have here?

S1 A multiplication!

T Ah, fantastic. Then we'll start!

S1 It's nine over two
(…)

T (…) but in this case: what is the line now? In algebra.

S2 A divided sign!?

T Yes (…) so what is the first algebraic rule? Give me one! (…)

S3 No times sign

T No times sign, yes. (…) If I have a construct like this here, of course, that in between yeah, always think that the three, if I would now tear the three and the x apart, then the multiplication sign would be, would pop up. But we squish it together and that's why we can't see it. (…) next rule!

S3 No division sign

T Yeah. How does the division sign look like?

S3 Like a fraction

T (…) Next rule! (…) is that correct?

S4 Number goes before the letter?

T Yes. The numbers go before.
And what is with the signs? Which sign belongs to which term?

S2 The one in front, like-

T (27:50) Yes! Its always the one left to it.
(…) That you write down the terms on cards (…) These terms all have names. What is the name of this term? How is it called? You can see it.

S4 X, eh, the last, the letter x, like, ehm

T Here, what happens here? x squared term, ok, so, you see the letters, and what is on the card is the name of the term. So this is the x squared term. What is the name of this term?

S4 Number letter?

T No, only, what's the name of the letter?

S4 x

T It's a x term. Here, we don't have a letter. What is the name of the term?

S5 Number term?

T It's the number term. The name of this term would be? (.) only the letters

S5 Xy term

T Ya. Xy term. What is the name of this term?

S1 T term.

T Yeah (…) Why do we have to know what the names of the terms are? Why is that interesting for us? (…)

S2 When you collect like terms?

T Yes, when we simplify. Because what are we only allowed to do when we simplify (…)

S2 Collect like the same -

T yeah

T (29:55) Here we have a x and here we have a x squared term. Both terms have a x. Are we allowed to collect them? Or are we allowed to put them together?

S2 No, because one has a b?

T No!?

S2 No, just, no, I say no

T What is the name of this term?

S2 X squared term?

T Aha, and this is a?

S2 X term?

T Do they have the same name?

S2 No

T No. So are you allowed to collect them or are you allowed to put them together?

S2 No

T No. because: they don't have the same name. they're not like terms.

T (31:50) Yes, distributive law. (…) Now, how do we solve it?

S2 You have to do a times b plu- and a plus c

T Yes. So both terms in the bracket is multiplied by a. (…) can you remember? We drew it down, we showed it as a picture, for example, as a rectangle, one side is a, and I split up the width into two parts, one is called b, and this is the length, this is a, and this is c. And I want to find out the area of this rectangle. What do I have to do?

S5 You have to multiply a by b

T Yaha (.) then we have this

S5 And then a times b plus c minus b, or?

S4 No, just a times c

T So, from here to here, it's b. So to find out this part of the rectangle, it's a times b. Now, we want to find out this part

S5 Yeah, it's a times b plus c minus b

T	No, because b is only here
S5	Then a times c
T	Yes, a times c. And now I want the total of the whole rectangle. So what do I have to do?
S4	You plus them
T	You plus them, so you have to plus this area ab and you have the area ac and you add them up, and you have the whole area of the whole rectangle.
S8	So the area would be ab plus ac?
T	Ya. But then, the area is always with the square it sign, so would you- ? (put it with) a squared sign? I mean if it would be centimetre, would it be like square centimetre?

T (03:55)	What do we do?
S1	You put the numbers instead of the letters
T	ok, we substitute.
S1	eh, that would be, eh (.) ehm four squared
T	yeah
S1	in the brackets is minus two plus minus three. And then you do that. Four squared times minus five which is minus eighty
T (25:03)	(…) because it is not a tick, it is a root sign. (…) And this means that everything that follows is in the root. Now, what was the root again? Can somebody remember what that was?
S2	I don't really know how to explain it but isn't it like, kind of like squared or cubed just like reversed?
T	yes! It's the reverse operation. One of the reverse operations. So, in this case, it is the square root. (…) There is normally a small tiny two but we don't write it down. So if you have a root without a number, it is a square root. I f we have a three here then it is a cube root. What do I do first?
S2	Ehm, you plus them … so it's

T	How to I plus a squared and c squared?
S2	Oh no, you have to conv(ert) them into numbers?
T	you substitute.
S2	ya (…) it's sixteen – no – yeah
T	So a is four and c is minus three. So what do we do?
S2	It's sixteen plus nine? Minus nine?
T	that's the question now! What is minus three squared?
S2	it's- I think it's minus nine. It's nine!
T	What is minus time minus?
S3	plus
T	plus, ya, so it is positive, so we add nine. So this is a tricky- because first of all, you- the exponent comes first, so that's why minus three times minus three, that's nine. And then we have a plus sign anyway so that's why it stays positive. Ok, then, next step is?
S2	it's twenty-five
T	And now, mmh, what is the square root of twenty-five?
S2	five?
T (35:25)	Is five a square number?
S1	no?
T	no (…) So, if you look at the examples before, we always had a square number in the root. Here, it was a one and here it was a twenty-five. So in this case, you can extract the root. (…) So normally, you have a positive and a negative answer. (.) In this case, the five is not a square number. And if you would extract the root, what kind of a number did you have, Levin?
S4	two point two three six and then some other digits
T	Yes, so, if we would now extract this number, then you would have a decimal and it would be a decimal that, first of all, never ends, and is, eh, there is no pattern, yeah, so it is not that we would have something like six six six six six or one two three one two three one two three. So, the answer would be in this case a non-recurring, never ending, decimal. So, and these numbers are called irrational numbers. Yeah? Because we have rational numbers and irrational numbers. And irrational numbers are numbers I cannot – so, actually every

	number I can display as a fraction. Ya? And these numbers, I cannot display as a fraction. So that's why I can't even – and –so, that's a little bit difficult – numbers is – normally, we want to work with numbers that are exact. Because at the end, we want to have an exact, ehm, result. So, and in this case, if I show the root of five in this way, then it is the most exact way to show root of five. As soon as I extract the root, and I would write down the decimal, then I have to, at some point, have to round the numbers. So I have to cut it off and shorten it because I cannot write down a number that is non-recurring, and never ending. That doesn't work out. So, and as soon as I round a number, then the number is not exact anymore because it is not displayed in a way it is because now, I cut something off or it is not exact. So that's why if you have a root and in the root is number that is not a square number then you do not extract the root. You leave it in this way and you're finished.
T (44:45)	(…) When we have these different terms, that we had last week, that we can give them names, for example the x term, the x square term, the number term, the x y term, the whatever term, ya? That, we can simplify the terms. What does that mean? What do I do when I simplify?
S2	We collect like terms?
T	yes, you collect like terms.
T (46:13)	(…) and the instruction is simplify as far as possible. So, Nicholas, what should we do?
S1	It would be x equals ehm seven minus four.
T	See, that's a difference. They say simplify as far as possible. They didn't say solve for x. So that's a difference. They want you to simplify
(…)	
S5	Yeah, you have to look at all the terms that have the same letter? So Y is two times and x is once. (…)
T	yes. Imagine, if you would write these terms on cards, the sign left to the term belongs to the term and when you would use coloured cards, you would see that this here is a y term, this here is a y term as well and that's why you would put them together. And then it would be Gulina?
S5	eh, eleven y and three x?
T	yes.
T	(…) So these are like terms. And the others?
S6	seven b and plus b
T	so, are they like terms?

S6	oh, no! I'm – they're both – eh – different terms.
T	yeah, why?
S6	because the one is – eh – has a number and the –
T	does that matter? (.) No! So the letter, the letter gives the name. So it doesn't matter if there's a one hundred fifty in front of it or a two hundred – the letter gives the name. so, in this case, these two are b terms because they only have a b. and these are a terms.
S7	Does it make a difference if there is a b squared?
T	yes, then it's a b squared term. Because the difference is, how it is, because you have here a b. and a b is a b. a b squared is a?
S8	b squared.
T	no – long form is?
S8	b times b
T	yes. See, and this is the difference. This is one b and this is a b multiplied by itself.
S8	ah, ok.
T	It is also a difference if you have centimetres or centimetres squared. It's not the same. Why?
S8	because centimetre squared is centimetre times centimetre.
T	ya, so this is a length and this is a area. So that's why you cannot put them together as well. They're different units.
T (06:40)	What are they called? What's the name of the term? Number term. Because they don't have any letters.
T (08:25)	So, how many terms do we have in number seven?
S9	Eh, in number seven: we have (…) three!
T	great. What are their names?
S9	eh, a x term, y term, x squared.
T	(…)
S9	you minus the two y with the f- wait – (.) negative t- I mean – two y and negative four y?

T yeah.

S9 that would be negative two y?

T mmh

S9 And in the front comes five x!?

T in the front? No, here! Yeah?

S9 and x squared.

T is it only x squared?

S9 (.) negative x squared.

T negative x squared. And what comes in front of the five x?

S9 positive? A pl-

T positive. Coz' here is no sign. That means it's positive.
(.) First, x squared because the exponents are- have the- more value, yeah? Then we have the normal x and then we have y. So the terms in the alphabetical order.

T (02:00) So what did you do?

S1 I did four over x plus one over x.

T so these are like terms. Why are they like terms? How did you recognize?

S1 they have the same x

T yes, because they have an x in the denominator. And, so when you put them together, you have? How many?

S1 five over x.

T (…) what did you do then?

S1 minus seven over y and plus two over y.

(…)

T (04:45) How do we solve this? Is there something new or- ?

S2 First you have to do the brackets, so-

T First we have to do the brackets?
(…)
Wait a minute, we have a bracket, we have a multiplication and we have a addition. So when we see this, when we see a equation or a number problem that has different operations and a bracket, what is the first thing we think about?

S3 Bidmas!
(…)
T So, first of all, it is the brackets. Can we add x plus one?

S4 ya

S5 no

T no. So, how do we get rid of the bracket? What do we have to do?

S5 the two times the x and the two times the one.

T and how is it called again?
(…)
S6 distributive law
(…)
T So, what you first have to do is that you get rid of the bracket by using the distributive law. So the two in front of the bracket, even if the two is behind the bracket, we know that in between the two and (the y) if we would push them apart, what would we see? What would appear? What don't we write down?

S2 a times sign

T a times sign. So it is a multiplication.
(…)
Ti so two times the x is two x and two times one is two

T so plus two

Ti ya, then you add up the two x and the thre-

T first, wait a minute, what do we do with the three x?

Ti (.) eh we leave it like that

S2 three x plus two x

T so, we put it down, so, and what I would advise you, the terms you did not use, or you did not calculate with, that nothing happened with, you write it underneath, yeah, so don't put the three x now at the, behind. Because sometimes you have a minus or here is a negative, that, if you don't put it in the correct order, that, when you calculate from left to right, the you answer is wrong. So the terms you haven't touched, or nothing happened with, write it underneath. That you keep the correct order. What do we have to do then?

Ti	add the three x and two x to five x and you put the plus two down and it stays like that!? And that's our answer.
T	ya, perfect. So it is five x plus two because nothing happened with our two.
(…)	
S6 (08:40)	so it's five x plus seven x minus seven.
T	ok, what did you do?
S6	so, since we don't touch the five you just write five x. plus. And then it comes to the brackets which you have to times. So open them up. What that means then seven times x, then seven times one.
T	ok, then it is seven x
S6	= seven x, yes, minus seven (…) And then it's, you plus five x by seven, which is
T	oj, what is that?
S6	yeah, you plus them!
T	oh, ja, I understood times
S6	(twelve, ya) and minus seven
T	coz' nothing happened.
(…)	
S3 (12:00)	minus what? Minus what?
T	what do you think?
S3	seven x minus?
S7	nothing
T	if it would be nothing, so, common sense, which number would be in front of the bracket?
S7	zero?
S8	one!
T	if it would be zero and we would multiply it out, what would be the answer?
S3	x minus zero

Stds zero

T it would be zero. Does that make sense?

S3 no- then it's one!

T so it has to be one, ya!?, because the zero doesn't make – because zero, you would write down because the answer would change. If you have a one in front, like that, then the x and the minus three don't disappear. Or the terms don't disappear.

S3 then its six x plus three!?

S7 plus, ya!

T first of all, what happens with the seven x? in the first step? Nothing. So we write it down. And now, what do we do?

S3 negative one times x

T and that is?

S3 that is negative x

T negative x.

S3 and plus three, negative one times (minus three)

T yes, and this is the important thing, that the sign in the bracket changes

(…)

S7 Why do you do then plus three and not minus three?

S3 because negative times negative is positive.

S7 and why negative times- ah!

(…)

T (15:15) so, my question is: is y times zero correct?

S9 it's y times (.) like, like, no!

(…)

T no? why?

S10 (?) the one in front of the second bracket

T ok, first of all that.

S9 and then you have to work out the first bracket, write it down, second bracket, write it down

S3 and you still have to multiply the y from the brackets

(…)

T	so, of course, we have to multiply them out
S3	so why times three y is four y? no!
T	times?
S3	because I thought the y have to-
T	do you add? You have three ys and you multiply it by y
S9	we don't know what to multiply it by.
S10	three y square!?
T	ya: three y
S9	square
S3	and then y times minus one is one y!?
T	yes, positive or negative?
S3	negative?
T	yes. So minus y. and always make sure that at the beginning, please use arrows, that you don't forget the second one, because mostly, it is that you multiply out the first one and the second number, is a typical mistake, then they write down, minus one. That's wrong. So always keep in mind: both numbers in the bracket have to be multiplied. Ok, now we have that. And how would you multiply the second one?
S3	because there is a one, it would be minus three y and one?
T	and now: is it positive or negative?
S3	negative, or?
T	yes, because of minus times minus. (…) and now, the typical question: is y squared the same as y?
Stds	no.
T	no, because y squared is y times y and this only a y. it's a difference.
S3	is it three y squared minus two y plus one?
T	mmh, plus one is ok but are you sure that it is minus two y because we have minus y, minus three y.
S3	no, then its two y

S9	four!
T	you have a credit of –
S9	minus minus
S3	ah, then its four y
T	because I give you a y, ya?, so you owe me one y, and then I give you another three ys. So how many ys do you owe me?
S3	four!
T	So that's why minus four y
S3	and why minus?
T	because how many do you owe me? It's a credit. Because I gave you one y, so you owe it to me, it's a credit, so it's minus, and then I give you another three ys, so you owe me another three ys. So, in total, how many ys do you owe me?
S3	ah, I got it.
(…)	
T	now, two brackets (…) who can remember how to solve it with the two brackets? How do we do it?
S10	ehm (.) you (.) the times one each bracket and then you solve them both
T	how? Just- how would you do it?
S11 (21:25)	first x times x, and then x times three, then five times x
T	ah, ok, you got it
S11	and then five times three
(…)	
T (22:35)	now, the tricky thing is that you have two brackets and in this case the x has to be multiplied with x plus three and the five has to be multiplied with x plus three. So, this is a possibility that you use the arrows to make sure that you multiply each letter or number with the other. Another possibility is that you write it down in this way, and you have to find out your perfect method, is that you know, ok, x has to multiply the whole bracket, so you split up the bracket. So that you say, ok, x has to be multiplied with this, and now I have to multiply this bracket by five as well. Like that.
S3	is this the second method or?
T	ya. So this is the first method, this is the second method,

ok, so you split up the first bracket, then you have in the second step, you have two times the distributive law. Because you have x times x plus three and five times x plus three. And then as we did it beforehand, you multiply it out, as we already did, so that would be (…)

S12 (28:15) two x times x (…) is x squared. And two x times minus three which is six x

T mmh, positive, negative?

S12 positive?

T you have a positive here and a negative there

S12 so it's negative!?

T it's negative, ya, and then it's six x, right?

S12 then you do one times x which is x (…) and one times minus three. Is negative three

T yes, good. And then the last step, just simplify

S12 ya, so, so its two x squared minus seven x

S13 minus five x

S12 oh, ja, minus five x minus three?

S10 (29:45) you write two x, then in brackets x minus three, plus one, in brackets x minus three, then you do two x times x which is two x squared minus six x plus x plus three – minus three!

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S1 (05:05) (I have eight)

T it's eight, you round down

S9 But don't you round to nine?

T no. it's always when you have something- when you have eight point four nine and you have to round to the whole, this one is the decider. Always the unit next, beside to

S9 oh, I thought that you round to nine, which, so it would be eight point five and (that round to)

T	yeah, no, you, the decider is always the unit right beside it.
S9	ok
T	(…) I learned it at school that you started here and then you rounded (…) but its wrong. We don't do it.
S2	it's unlogical because if you round it up then, like it's fifty-one digits from the nine and forty-nine from the eight
T	and that's the argument. Because if you would look it out on the number line, then it's forty-nine is nearer to the eight and it would be fifty-one to the nine. (…) so, the unit right beside is the decider, it tells you what to do.
S10 (06:50)	is it also ok if I write seven hundred?
T	no, you have to write the whole number. You're rounding the whole number, guys. So it is that when you have twelve thousand six hundred seventy-three, they want you to round. So you have to think as if, the number line again, they ask if you would see this number on a number line, ya, it would be between twelve thousand and thirteen thousand. Now they ask you, when you round, to which thousand is this number nearer?
S10	uhm, to thirteen thousand.
T	yes, so round about. 'coz here would be five hundred, and so it would be here. And that's why you round up to thirteen thousand. Not t(w)o thousand or three thousand. It's the whole number. So don't cut it off or don't make it smaller.
S10	'coz I just wanted to write seven hundred because the whole number is-
T	ya, no, it's the hundred, like this is: these are the thousands, so if you would have a number line jumps of thousand, and now you would look at, the next task is, you look at with jumps of hundreds, so you would have this here: twelve thousand six hundred, twelve thousand seven hundred. Between, so it's between these two hundreds on a number line, and to which one is it nearer? And in this case: seventy, so it's more than five, uh, fifty, so it's roundabout here.
S1 (10:15)	so you always have to write thousand hundreds and (?) when we round?
T	no, it's nine thousand here
S1	ya, here it's nine thousand but-
T	yes, and here as well because here, the hundred, the next hundred is the next higher thousand, look, you have eight thousand nine hundred seventy-three. (…) They said: round to the next hundred. And if you look, you have, you have

	nine hundreds. And then, the confusing thing is you have nine hundreds and you have to round to a hundred. And then most of you didn't know what to do. Think of a number line! So if you have jumps of hundred, it would be eight thousand nine hundred, and the next hundred is what?
S2	nine thousand.
T	nine thousand. So, you have to round up to the next thousand. So, it is the-, we don't have a hundred but it already starts by the nine thousand. So you have the next thousands and you start with the zero hundreds.
S11 (12:00)	b: one!?
T	one what?
S11	no, one thousand one hundred
T	uh uh, you have to round to the nearest thousand
S11	uhm, one thousand!
T (13:28)	then we have a school hall holds two hundred sixty-nine people. Round this number to the nearest hundred.
S12	two hundred because like they say before, (?) here like on the first page, it says: rounding (example three) (?) sometimes, you may need to use common sense (whether) you round up or down (…) so with the school hall (?)
T	oh, very good. (…) they said sometimes when you round, and especially with word problems, you have to use common sense. (…) Sometimes when you round, it is better. Even if there is a four and you would normally round down, or you have a five behind it and you would round up, sometimes it is not good to round up. For example, when they have this here, (…) the maximum height is seven meters because the bridge's height is six point seven. Is it a good idea to say the maximum height or- is seven meters? Is to round up? And in this case: so what would happen if my car is seven meters high?
Stds	crash
T	it would crash. (…) so that's why it is rather to round down, to say: ok, it shouldn't be higher than six meters, your car, because then you know there is enough space under it and you can drive through. And in this case with the hall, is the same: because they say: a school hall holds two hundred sixty-nine people, round this number to the nearest hundred. does it make sense to say, to put more people into the hall that fit into it?
S1	no.
T	no. so that's why in this case, it makes more sense to round down

S2	but they don't say how many
(…)	
S13	but why do we like round it down, like two hundred, because then they don't fit any more in, because if you say three hundred, then more people would fit in
T	you have to imagine, in this hall, so the fire station says, it holds, or, let's take the elevator, that's easier. It holds the weight of two point five six tons. Now, they just want to know: how many tons? Is it now better to say: well, roundabout three tons or roundabout two tons?
S3	two.
T	two! because if it would be more, then the elevator would crash down. (…) how much are you allowed to let in?

T (20:45)	(…) is sequences, that we have two magic words. Which are the two magic words we have in this context?
S1	term to term rule, and the
T	what was the other one?
S1	the (.) the nth term.
T	the nth term. Good. (…) now, term to term rule. What is the term? Who knows it? So first of all, I'll write down the sequence. The easiest one we have: (…) we'll do that: three, five, seven, nine.
S2	uhm: two n plus three or three n plus two, no, no, no, no. two n plus three.
S3	no
T	not yet, no.
S3	three n plus two
T	three n plus two? no.
S4	two n plus (one)
S3	no, right
T	two n plus one? What now?
Stds	two n plus one!
S1	two times one, plus one is three.
T	ah! So, what is first of all then the term? In the sequence?

S10 which number (?)

T ok. So, this is which term?

S10 the first term

T aha. So, we remember: the first number in the sequence is the first term. Then the second one is? Term two.

S11 (23:40) and nth term. (…) It's easy. You just have to find out the difference between (n) so then and the number before n. two n. and you just have to find out (?). two n is always three plus two is five, five plus two is seven. So you have two n, that's always the first question and then you plus one, so it's two n plus one.

T so first step, what we do is, what is the difference between the numbers!? It is two. we always add two. so that's why we have, we put the two in front of the n. what is n?

S5 number

T which number?

S5 a, a number.

T yes. (…) so, two times one, so the one is in this case term one. So two times one is two, and if you look at the term one, what is the number? Three. So, what is the difference between

S6 one

T so if we write down the multiples of two underneath, because it's, we always add two, we can see the difference. And the difference is one. Do we have to add it or do we have to subtract it?

S6 add it

T yes, because it's more. So we add it, and this the second part of the nth term, that we? Add it.
So the second key word is the position to term rule, and this is what they love as well. 'coz normally they give you a sequence, then they say: write down the next two numbers in the sequence (…), then they ask, maybe, what is the nth term of the sequence or then, you would have to write the nth term, so in this case, we would write t n equals two n plus one. And now, normally the third question is that they ask for the position to term rule, that means, which number would be in the tenths position? So which number would be the tenths term? What do we just have to do?

S14 we have to substitute ten for n!? and then we just have to multiply it out, which is twenty-one.

T	what would be now the fiftieth term? What do we have to do?
S9	you times two by fifty.
T	ya, so two times fifty
S9	plus one?
T	plus one. And that would be?
S9	uh, hundred (.) plus one. Is one hundred and one.
S7 (28:26)	so the numbers in the bracket, is what you add up, or?
T	no, what is the number in the bracket? (…) the number in the bracket is the term you want to find out. So, here is the nth term. If I want to find term one, then I put one in here. And I substitute for n a one. Here it is, when we write down the sequence, (…) all the numbers, and now we want to find out the fiftieth number in the sequence. Which number is it? So n would be then five, uh, fifty, because it's the fiftieth position in the sequence, and then of course the number of the term is fifty, and that's what we substitute.
S7	but because the calculation is-
T	What is between the two and the fifty? why do I put-
S7	times.
T	times. That's why I put the number in brackets.
S7	so two times fifty plus one.
S8	like the two n plus one, is that now the term to term rule or the nth term?
T	the nth term
S10	so if they write in the test: find out the tenths or the fiftieths term of this two n plus one, we just do: t – so, you just do two times fifty plus one? Or two times fifty plus four?
T	why plus four?
S10	if they change it
T	yes, so what you just have to do is, you just replace the n with the number of the term you want to find out. Algebra: we substitute

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T (01:50)	what is a term?

S3	number!?
T	what number?
S3	the-, of the nth, the, the, the term, the first term (?) it's always the next number, or the number after
T	so, in a sequence, we have terms, and how do I find the number or which term does the first term have?
S3	we always have like, like three n minus two, and then you have to calculate it.
T	yeah, but how do I find the first term in a sequence? What is a term?
Stds	one? Four?
T	and the third one is?
S3	three?
T	aha! So you have the terms: term one, again, you have in a sequence: the values are the terms. The position is the number of the terms. So the first one is number one, the second one is two, the third one is three, four, five. So, number n, or the letter n, in the formula, is the term, is the number of the term, it's called n because of the word number. How do I find out the first sequence?
S4	it's like one, three times one minus two.
T	is?
S4	one.
T	ok, the second one is?
S4	four, seven, ten
S3 (06:10)	ok, it's like two n minus one half. It's two times one is two minus a half is one point five.
S1 (07:40)	I just had problems with the, with the n, like-
T	ah, so if you have this here, something like three point five minus zero point five n, please, if you have this, of course, you never turn around the order. So in this case, if we would want to find out the first term, you keep three point five in front and the minus sign as well. You're not allowed to put this answer at the beginning. So, first of all, of course because it's a multiplication, the multiplication first. So zero point five times one is zero point five. And then

	you have to start from the beginning. Three point five minus zero point five. And then you have it. Then term one is three.
S1	so (then one is) just times one? Because here it's like eighty minus four n, so it's four n times one, which is four and
T	ya, ya, 'coz it's always keep the order. Don't change it.
T (09:35)	(…) or they want you to find out the sequence by looking at the picture.
(13:55)	(…) now, in the next pattern, you see that our first pattern has been extended by three counters. (…) so we have seven, the second one we have ten and then it is extended again by three dots so it is thirteen. So first of all, now, because what they definitely want you, if you look at this, that you create the nth term. So what is the first part of the nth term?
S5	three n
T	three n. why? How do you see it, that it is three n?
S5	'coz you always go (up) by three!?
T	ah! (…) three n because we always add, in the sequence, we have always three dots more in our picture. Now, how de we find out the second part?
S6	like, uhm, I mean, three n plus four!
T	why plus four?
S6	because we have three and like and uhm start at seven and ya: three plus four equal seven
T	and you can see that you have to add four. (…) now, we have three, the difference is four and that's why you have to add four. The same thing here: because here is the first term, the second term, so if you would times two by three then you would have six and you would have to add four again to get ten.
(…)	
S1 (16:40)	if we know it's three n then we just have to find out the difference of the first term and the n.
T	ya, so what you do is: the first part, again, the number in front of the n is how many you have to add or subtract. You write down the multiples of this number; look at the difference, this is the second part.
T (20:02)	now, again: if they say: match, it's similar a little bit to memory, ya!? That you want to find two pictures, or in this case three pictures that are similar, or that they are identical. So you have at the beginning the pattern as a picture, in the second column they say which number each, or the terms have!? Ya, so in case,

if you have t in brackets one, it is the first term; if you have t in brackets five, it is the fifth term. And the third column, if we look at it, is the nth term. Now, if we look at a, we have the first, we see always the first three terms. How many dots do we have?

S3 a, we have seven!?

T seven dots-

S3 nine, wait (?) ja nine, and fifteen.

T no that can't- no! here in the third picture you don't have fifteen. That's not true. Be careful!

S3 [counting] eleven

T aha! (…) now: this is what you see on the picture: the first three terms and you can count the dots. Now, you look in the second column, and for example, i) says: the first term has seven, the second term has nine, and the fifth term has fifteen. Does this match? With this picture?

S1 ya

T why?

S1 because (they give you like) the first term, it's also written there: it says seven dots, the second one has nine and if you would go on with pattern (?) like then it would be thirteen and then fifteen.

T aha. So: one, two, three, four, five: this is the fifth term. (…) again: on this picture, you see the dots. So, you count seven dots, you count nine dots, you count eleven dots. Here, in the column here: it says: the first term has seven, the second term has nine, and the fifth term, this is the fifth term, has fifteen. One, two, three, four, five. Ya, so: a is the same as i. now, if we look at the sequence: how does the nth term look like?

S7 two n plus five?

T why two n, first of all? Because-?

S7 well, (the term uhm two n and then plus five)

T yeah, but why is it two? try to explain that you keep it in mind. Why is it two n, why the two? how do you see it in the sequence? Because? What's always- because we add- is? We always add two. that's why the first part is two n. ok? Then you see it already, it has to be two. but why plus five? That's what you have to know as well. We have a two here, and if you write underneath the numbers, the multiples of two, what do you see?

S7 plus five.

T	aha. And that's why this is the second part of the nth term Now, look at b): how many dots do we count first? How many dots?
S8	the first is: eight
T	ok, eight. So, eight dots in the first picture, then in the second picture?
S8	eleven.
T	eleven dots. And then in the third picture?
S8	twenty.
T	twenty. Sure? Never! Ya, you have to be careful. The third. The third picture. How many dots?
[mumbling]	thirteen, fourteen, so! Fourteen dots. Now, this is, again, this is term one, this is term two, this is term three. Now, we look at the second, the rule, which terms are identical with these ones? With this picture? Is it i, ii, or iii? so which one? (.) we started with how many dots?
S8	with eight dots
T	aha. So now, look, we have: term one is eight. Term two is?
S8	eleven?
T	eleven. Term three is fourteen. Term four is?
S8	uhm, seventeen
T	seventeen. And term five is?
S8	twenty-one
T	aha.
S8	no twenty
T	twenty. Ok. So we got this. So this is iii. now nth term, how does it start? When you look at the sequence, how does our nth term look like?
S8	eight- uhm
T	uh uh. Look at the diff-, look here: it was seven, nine, eleven: two n. ya, because: two n-
S8	three n!
T	aha. Three n. then, we write down the multiples of three underneath: three, six, nine, twelve, fifteen. So what is the second part. Look: multiples-

S8	plus five!?
T	aha, and here it is? Plus five as well. So it's the roman number three.
T (28:40)	now, we have this picture with the matches. And you have to read. It's more the reading thing, I think. Because they say: these hats had been made with matches. And you see the first three terms. Then: a rule to find how many matches are needed is: the nth term equals five n plus one. So with this formula, you can find out how many matches you need depends how many hats you want to have. So, now, they say by a) use the rule to find how many matches are needed for twelve hats!? (…) you just use the formula. And again: the letter n stands for what?
S1	number.
T	which number? The number of what?
S7	(the hats), so the, the
T	the term, ya!? So, they want you to find out the (twelfth hat) so: number, or the letter n is which number now?
S9	twelve.
T	ah! Now, we want to find the twelfth hat. So n is now twelve. Five times twelve, and not n anymore,
S1	plus one
T	plus one.
S1	equals?
T	so, five times twelve is?
S7	oh, five times twelve is sixty
T	plus one is?
S1	seventeen
S7	sixty-one
T	this is all! Now: I use eighty-one matches to make some hats. How many hats do I make? How can I find it out?
S10	you, uhm, subtract one from eighty-one
T	so, first of all, what do we want to find out? How many hats?

S10 yes!

T so, do we know the number, the term of hats, the number of hats? Yes or no?

S10 no.

T no,. so we leave it as a n. because we don't know the number of hats. So that means the formula stays the same but what do we know now?

S10 how many, uhm, how many matches we have.

T ok, and that is?

S10 eighty-one.

T eighty-one. So, what do you do now?

S10 (31:30) you go exactly backwards. Which means you, uhm, subtract one from eighty-one

T and again: now, we are coming to these equation thingies what we did last week. So you do, on the both sides, the same. That's why you subtract one. And eighty are left over. What do you do then?

S10 you divide both sides by five. And the answer is then sixteen.

T so you do everything the reverse. That was good!

S11 but I also had a other way how to find out the-, the term number

T what did you do?

S11 when I did number, what was it again?, well, ya, the eight, I think, I found out that the first, that the ten, that the digit of the ten in sixty-one is the, well twelve is the double of the six, so eighty-one, the double is sixteen

T ah, you saw the pattern! Look: she looked at the tenths, here is a six and she saw: she multiplied it by twelve. So she looked at the tenths at the answer here: it's a eight and then she doubled it as well and that's sixteen.
But it not always works, ok!?

S11 ya, not always. It was just an exception.

T (…) She just looked at the tenths and saw: she multiplied by n: was the double of it. Now, she knew here: she has eighty-one- so she looked at the tenth: eight. And then tried out and doubled, doubled the number, and n is now the double of eight: sixteen.

(33:50) c): different hats are made with matches. We have the first hat with five, the second two hats with nine, the three hats with thirteen. This, again, even if it

	annoys you, these three pictures are the first three terms in the sequence. Now, what they want you to do is: find the formula that matches with the picture. Which one is it? First of all, when you again look at the picture, what is the first part of the, uh, of the formula? What, how does it have to look like? The first part of the formula? We look at the three first numbers of the sequence. Then how does the first part of the formula look like? (.) again, we have five, nine, thirteen
Stds	four and five
T	what's the first part?
S10	wait, what do you mean?
T	the first part: we have the blablabla n plus minus something. So what is the first part of the formula?
S1	it's four n
T	yes, why?
S1	four n plus one.
S2	the difference between the terms are four.
T	ok, so we add four. And that's why it is at the beginning four n. and what is the difference?
S2	one.
T	ya, ok.
S2	ya, four n plus one.

T2 (37:15)	(…) number of blue tiles. so this two.
S11	ya, that's two, then three, it always increases by one. Then, at the third term, here: there is four, then again it increases by one, for the fourth term, it's five tiles then for the fifth, it's six
T2	ok, what did you see from pattern, from the first to the second? The blues increased by how much?
S11	blues always increase by one
T2	and the whites always increase?
S11	by two.

T2	by two. ok, what does that tell you for the, for the nth term then?
S11	ah, we just should write the rule in there! I thought – ah!
T2	of course: how you get to the nth term, right!? So for the number of blue tiles: how should it start?
S11	uhm, so the nth term is (.) the nth term is: one n plus one, actually.
T2	exactly. Why the one in the beginning?
S11	because it always goes before the n!?
T2	yeah, but why, why in this problem? Why is there a one in front of the n and not two, three, four, five, six, seven
S11	yeah, well because, actually, you don't need a one, it's just n
T2	mmh, yes, but why is it a one?
S11	because, uhm, first term: you take it times one, well then, you have it, you have a one. But then, the plus one is two, and it goes on: three (?) yeah, n times three is – here it doesn't fit! Look: the number of white tiles
T2	right, right, then: there is a d- another rule, right?
S11	oh right, ya!
T2	so what is the rule for the white tiles?
S11	white tiles is: (.)
T2	look at the, look at the, see: pattern one: there are:
S11	two, then four, then six, then
T2	so: and then for the nth pattern?
S11	it always increases by two so it's times two: n times two!
T2	so just two n, right.
S11	ya: two n.
T (40:52)	now, we look at the tiles. Here is a sequence of tile patterns. We look at the tiles- (…) now: blue tiles. If we have now: this column, so tiles one, tiles three,
S11	uh: you mean term, not tiles!

T	or that. When we have the blue ones: how many do we have by picture number one? Two. picture number- by tile, picture number three?
S1	four!
T	four. Picture number five?
S11	six, seventeen and n plus one
T	uh?
S11	n plus one.
T	ah, why?
S11	because well n: you take n times one which is one and then you still add plus one, to pattern one, which makes two blue tiles!
T	yes. (…) now, when we look at the white tiles: how is the second column?
S6	so the first pattern would be like two.
T	the third pattern?
S6	six.
T	the fifth?
S6	ten
T	sixteen?
S6	thirty-two?
T	an how is the nth term?
S6	(.) n plus three?
T	no.
Stds	n plus one? N plus two? two n
S6	n plus two! n plus two!
S1	two n.
T	two n. why two n?
S6	ah: two n, ya! Two n!

T	look: two n because: one times two is two. three times two is six. Five times two is ten. (…) Now: look at the number of blue and white tiles for the general term and write an expression to show the total numbers of tiles in the pattern. Simplify your expression! Somebody find it out?
S12	three n plus one
T	whu! Good!

T (00:20)	so, the first square number is?
S5	two.
T	it's one.
S1	one times one is one
T	next square number!
S6	no – I – wanted to say something. one isn't a square number it's, uhm,
T	no, it's a prime number you're talking about. But square: one you can square
S6	oh, square number!!
T	yes, and not a prime number. Then, you're right. (…) so, one times one is one. What's two times two?
S7	four
T	ah! The next square. The next square number?
S7	six!?
T	eh.
S2	nine!
T	nine. Three times three.
S3	three times two or three times three?
S4	three to the power of two
T	the three, the square is when you times the number by itself

S5	sixteen
T	sixteen!
T (09:00)	so: today, it's just actually a very easy thought: is that we're gonna look at the difference between a linear sequence and a quadratic sequence. So, what we have discussed by now is, or we looked at (…) a linear sequence, are the sequences we looked at by now. We looked at, for example, t one was three, t two was five, t three was seven, and if we look at the sequence, or if we look at the terms, what do we notice?
S1	add five?
T	add five?
S1	uh: add two.
T	add two. and we always add two. ya, so the differences between the terms are always the same. And that's why it's linear because I always add the same amount or I subtract the same amount. Ya, so the definition of a linear sequence is then: the differences between the terms are the same. So, this shouldn't be a problem. We did that over and over. That we have the term to term rule where we would say now: it's add two; and the position to term rule or the nth term would then of course be two n plus one. (…)
(12:48)	now, if we have a new sequence, and you look at it and you would, for example, have this here: this sequence. And this is called, I already tell you: quadratic. So, if we now look at the seq- uh, at the terms, in the quadratic sequence, it could look like this: three, six, eleven, so: as you can see: it could never be a linear sequence because the differences between the terms are not the same. (Because as we can) see here, the difference is three and here, the difference is five. And if I would write down the fourth sequence, eighteen, and then the fifth one would be [seventeen]. So if we would write down the differences, it would be here: add three, add five, add seven, add nine. Now, here is the problem: how can we write it down as an nth term?
(14:45)	first of all, they could ask you in the test, if they would have the sequence: what is the term to term rule? Where we write down in words what to do. what would you write down? Without the nth term. just the step: rule, what do you have to do? Term to term rule.
S6	just write: add two.
T	does that work out? Add two?
S6	oh, I – I'm –
T	ya?
S6	plus three?

T	also not. It's difficult.
S8	add three, add five, add seven, add nine?
T	but then you have to- then you have to write everything down until never ending, because it has no end. Try to describe it a little-
S8	can't we just write the nth term?
T	no, not yet. It could be that they ask term to term rule. Then you have to, of course, don't write down the nth term.
S9	add three and then always two more
T	yes. Or add two more than in the previous term or something like that. Or in the book, they say: add the next odd number. Because we're only adding odd numbers. The interesting thing is now: what is the nth term? coz this is more tricky. And here, you have to recognize something, otherwise it doesn't work out.
S10	n squared plus (?)
T	n squared? N squared
S1	plus n
T	plus n. one times one is one
S2	no!
T	plus one
S2	plus two n.
T	plus two n?
Stds	no, no
S6	plus two!
T	just plus two, ya!? Ok, because here, you have to recognize that in the sequence, there are the square numbers. And that's why it's then n squared. Because you said something very important when I wrote down the square numbers, you said: there is already a sequence. There is already a pattern. So if you write down the sequence of the square numbers, it always increases
S3	by two more
T	ya, but now, if you look at it: it always increases by (.) it always increases by the next odd number. So: if you see, if have a sequence, of course they will not write down these numbers coz that's too easy, they would write down

	sequences like that: and as soon as you notice that it increases, or you always add the next odd number, then it has to have something to do with the square numbers. So it starts with n square
T (23:45)	so, the first term was two, then five, then ten and the last term was thirty-seven. What did you do?
S5	term one: seventeen, then twenty-six: add the next odd number.
T	ok, how did you recognize it? The first term: you added how many?
S5	three?
T	and then you added?
S5	five.
T	ok, so this one has to be then?
S5	seventeen and twenty-six.
S6 (24:35)	zero, one, three, six, ten and fifteen (…) Because from zero to one, it's plus one and from one two three is plus two, from three to six is plus three, and that goes on and on and
T	ok, and how did you explain it? What is your sentence?
S6	uhm, add the next number
T	add the next number!? Ok.
E (25:30)	because you always, like, add three to the number, like, if you have two, we add, so two plus three is five and five plus six is eleven, so we add three, so we, six, we add again three, and then we add nine,(?)
T	ok. So would you explain it in words?
S11	add three!?
T	uh, that doesn't work out. Because add three would always be always three. Plus three, plus three, you always: what did you?
S11	add three to the next number?

T yes, or always increase by or it's add three more than before, so, you start with three, first start at three and then three more

S1 I wrote: add the next multiple of three

T that's also nice.

T (27:00) what's the two missing terms?

S12 four and one

T and what's the rule? (.) So what happens here?

S12 minus nine

T and then?

S12 minus seven, (minus five)

T so?

S12 always minus two (then)

T so subtract!?

S12 ya,

T and then?

S6 subtract the- next odd numbers?

T but then you always have to at the beginning say: you start, ya, and then you. So, do you subtract two more or two less?

S12 two less.

T are you sure? Now, that's the question: two more or two less? (…) because here, you take minus nine and you take minus seven away. Is the number more or less? That you take away?

S12 two less.

S1 more!

T you take less away?

S1 you take more away!

S2 look: seven is less than nine.

T	ya, but you have minus nine and minus seven – no: it's correct.

T (00:07)	now, we look at it again. So: how do we find the length of one of the shorter sides. Now: more interesting. Longer sides is easy: the two smaller squares, we add them up, then we have the one of x. So, now: how do we find out the one of the smaller one? So, what is the largest square?
S4	one hundred and sixty-nine
T	yes, the next one? What's the area of this square? You have to learn them by heart. I said: never forget them. What is twelve times twelve?
S5	one hundred forty-four?
T	yes! So, we know that the largest one is one hundred sixty-nine and the one, smaller one is one hundred forty four. What do we now have to do to find out the smallest one?
S1	uhm, minus hundred forty-four from one hunderd sixty nine.
T	ya, but first one hundred sixty-nine minus one hundred forty-four. So we subtract them. So the left-over is then?
S1	twenty-five
T	twenty-five. And: what is now the length of the shortest side?
S2	twenty-five times twenty-five
T	so, the area is twenty-five?
S3	five.
T	oh, oh, the area is twenty-five. What is the length?
S2	five!
T	five. Because you have to find the number that you can multiply by itself that is twenty-five
(…)	
T (02:25)	again: take the pink area away from the yellow area, the left over is twenty-five, and to find – and you can see as how they write it down: x squared is equal to twenty-five. But we always want to find out x, so that's why we have to extract the root of twenty-five and then we find out x. because x times x is x squared and that's why five times five equals twenty-five. And here, you really have to be careful with the notation. You have to be sure, you have to know:

	what am I dealing with. Am I now talking about areas, then I have to write down x squared; or is it the length that I am talking about, then I have to write down x. ok, now we have found the length of x.
	To shorten the whole thing: (…) first of all, we always deal, at the beginning with the Pythagorean Theorem, we deal with the squares. So in this case: we look at the sides: how would be the formula then? How would you write it down?
S6	x squared is, uhm, ten, uhm, but why is there a two
T	why is it up to the power of two? because-
S1	because it's the area.
T	because here is a sign: which sign do you put? A plus or a minus?
S2	uhm, plus!
T	plus fifteen squared, ok. (…) what is ten times ten?
S6	hundred. And fifteen times fifteen is two hundred twenty five.
T	Then, when we add them up, we have?
S6	is three hundred twenty-five and then, we have to do, like, (undo that it's a square number)
T	now, you have to extract the root. This is now the case where you have to have a calculator
T (07:00)	here: extract the root: to extract the root is: you push the button root on your calculator and then it tells you that ten times ten is hundred.
S7 (08:05)	ya: eight squared plus
T	now: the other is?
S7	three?
T	three.
S7	(…) sixty-four and nine
T	together?
S7	seventy-three
T	ok, and when I extract the root? What is the answer?

S7 that is eight point five four four zero

T one d p

S7 eight point five.

Class level: Grade 9

August 2013

S1 square numbers!

T ya, seven times seven. How do we call the numbers with two same factors? What's the word?

S1 (?)

T square numbers. So, fifty-one: prime or not?

S4 yes.

T uh, multiple of three

S4 it's not! It's not.

T (00:50) twenty-one!

S5 no.

T no. what kind of a number is twenty-one? A multiple of?
(?) one and twenty-one and? Which multiple?

S5 (seven and three?)

T good. Twenty-seven?

S6 is not a prime number

T which other two factors does it have

S6 it has: seven!

T no.

S6 eight!?

T uh uh, can't be eight. Eight is a-

S6 oh, nine!

T nine. Eighty-one?

S7 no!? it is not!?

T it is not!? Why not? Which number fits in it? Eighty-one: important number. It is not a prime number, it is a?

S7	(square?)
T	yes! It is a square number. Because? Nine times nine. And what is with ninety-three?
S8	is not a prime number!?
T	why not? Which number fits in?
S8	it's not a prime number because it's odd and only the ninety-three fits in!?
T	no, then it is a prime number, if you think so.
S8	no no no I meant: no, it's a prime number.
T	the three fits in.
(04:30)	(…) and of course: when you simplify fraction and you have something like forty-nine over one hundred fifty-four, that you definitely know, ok: when it is the forty-nine, the only possibility can be the seven to try out if I can cancel it down by seven and if not, then I don't have a other possibility. So: the more you know about the numbers, the faster you are by cancelling down.
S1 (06:50)	a proper fraction is like, something like, when you have three over seven.
T	three over seven, so the fraction is? Compared: when I have it on a number line? Where do I find the fraction? Number line?
S1	I don't get the question.
T	here: look. For example: where do I find the fractions?
S1	between zero and one.
T	between zero and one. Is a fraction a number?
S1	(.) yes?
T	yes. Don't forget it. (…) so, fractions are numbers. Always always always, they just look weird. And we always find them between zero and one, and of course, on the negative side, zero and minus one. And of course, the definition is: here, we find the proper fractions, (…) so, a proper or a vulgar fraction is: smaller than one and of course improper fractions are larger than one. (…) and they look like, so they are equal or larger than one. (…) not only the improper or top-heavy fractions, which fractions are also larger than one?
S1	mixed numbers.
T	mixed numbers. And how do I recognize the mixed number?

S4	with a number at the beginning and with fraction in the end.
T	yes, so you have a whole number and a improper, uh, a proper fraction, so like two and a half. (…)
(10:15)	in the answer, are you allowed to leave a improper fraction? Is this allowed?
Stds	no, no
T	no. so at the end, always, always, always, make sure that your fraction is proper, and that you take out all wholes. So, you are not allowed to have a top-heavy fraction at the end. You have to take out the wholes, and have a mixed number at the end.
(10:42)	ok, addition and subtraction. If I have fraction plus minus fraction. What do we have to look out for? what's the first thing we have to think about? And this is always, even if you have letters later on. What is- can we just start calculating or what do we have to make sure before we add and subtract?
S2	uh, denominator(s need to be) the same with the (?)
T	yes, the same denominators (…) The first thing you have to make sure: so, if there are no same denominators, then you have to convert them. That's what we practised the last years, and then you are allowed to add and subtract.
(12:10)	Then, when we subtract or add, what do we do?
S3	(we add and subtract!?)
T	what? What do you add and subtr-
S4	we made the denominators – equal.
T	ya: same denominator. But when you have something like five eighths minus three eighths, what do you subtract?
S4	five and three.
T	aha. So, and what is that? What's the name?
S4	two over eight
T	no, what's the mathematical, how is the number called, up on top?
S4	nominator!
T	numerator.
S4	ah, numerator.

T	so, and this is what we had last year: is this then zero at the end or if we would have addition, would it be sixteen at the end? (…) is it correct what I did?
S4	no!
T	ah, what's wrong?
S4	because you have to - how is it called – to simplify it more
T	no- where is the mistake?
S4	ah, you added the denominators
T	am I allowed to add the denominators?
S4	no.
T	no! ya? That's what year eight did last year, constantly, constantly, they added and subtracted the denominators: awful. So: you leave the denominators in the way they are because the size of the part doesn't change. So first of all, you have to make it possible that you can add and subtract because the size of the pieces, you have to think, like, a little bit more visual, the pieces have to have the same size. Otherwise, you cannot add and subtract. If you take away or add, then the size of the piece, or the pieces, do not change. So that's why, we never ever add or subtract the denominators. They have to stay in the way they are. So we just only add and subtract the numerators.
S1 (14:45)	but if it's equal to one, it's not a fraction, isn't it? It's a number, like, uh, a fraction is a number but it's a full number
T	ya, and that's why it's improper. (…)
(15:50)	next thing is: mixed number plus minus mixed number. What do we do with the wholes?
S4	you change it to improper fraction
T	there are two ways. Which one is the easiest?
S4	in this case, then you transform them to improper fraction
T	ok. So you convert them. Why? Would you do this? Would you always convert?
S4	you get rid of two and three and then
T	do you always do that, that way?

S4	I do. If it's this way, yes. But if it was like two (with half) plus three (with half), then I would do, I would just add together (?)
T	so you would always convert into a improper fraction!?
S4	not always. If it is, if it has the same denominator, then no.
T	could you do it?
S4	this?
T	if you would do this as the first step, same denominators, how would it look like?
S4	you just add them. If it has the same denominator,
T	ya, but convert into the same denominator. Do the wholes change when you convert them into the same denominator?
S4	no
T	no, the wholes stay the same. Nothing changes. So, and if you would convert it, it would be six of course, and then we have four up here and three up there. And then you wouldn't?
S4	then I wouldn't.
T	for the ones now: the easiest way what you can do is: at first, is, the first step what N. said: first, turn it into a top-heavy fraction. Because by addition and subtraction it is actually not necessary, you do not have to do it, because I don't do it – I do it the way N. does it, I add the wholes. This is possible, if you are not sure, then always convert the mixed numbers into improper fraction, put them into the same denominators, and then add or subtract them. This is the easiest way because we always do same strategy. Ya, if you think it's easier for me first add the wholes, it's totally ok.
S1	so, if they now give a proper fraction like three over eight plus five over eight, and then your answer will be like eight over eight. Then we have to convert it directly to a mixed number?
T	this here?
S1	because: you can't leave an impro- uh top-heavy fraction-
T (19:40)	no: never! Here, you have to write down one. Otherwise, this: I would never accept. (…) and in this case, it is really: to turn it, that's what I would suggest, for the ones that still struggle, convert mixed numbers into improper fractions. So, the first step, N. said. How do I turn two and a half into a improper fraction?
S5	you take two whole sixes because like the two, like one

T	(…) how would you turn it around?
S5	you just add two times two
T	ok, two times two is?
S5	four
T	and then?
S5	it's five over two. because you add the number you get with the top, with the numerator.
T	so here: you multiply by two, and then you add one. I show you as a picture: you have two wholes and a half. And what you do is: that, you of course divide each whole into two parts, that's why you multiply. Because: you have each whole, then you double the amount of parts, so that's why two times two. then you have four parts, you already have one, that's why you have to add. Then you have five over two. that's the system. Same here: you have three wholes and two over three; and in this case, we divide each whole into three pieces, or three parts, not pieces, so we triple the amount; then we have, if we count them, nine, we add the two we already have and then we have nine over three, then we have fourteen over three. Am I finished? Did I forget something?
S5	you had nine, you just
T	ah, eleven! Thank you
S5	ya, that's what I meant.
(…)	
S8	but why over three, and not over two?
T	because here, the fraction has three; here, the one-
S8	- but the other one has two
T	ya, (.) thank you. (…) so what do we have to do first? Can we add it?
S8	nee,
T	no, see, now I was brain dead, so: what do we first have to do?
S8	convert it into (another number)
T	what? Convert it into?
S8	ah! Find the same numbers

T	yes, so that's the next step: again: same denominators. ok, what would be the next multiple?
S8	so: two times three because, that would be six on both
T	ya,
S8	and then five times three is fifteen and then eleven times two is twenty-two.
T	good, now, we can add them. That's thirty-seven. And now the question again: am I allowed to leave it this way?
S1	no
T	no. what do we have to do?
S6	(?)
T	you would take out the wholes!? And how many wholes would you get out?
S6	six and-
T	how many parts are left over?
S6	one
T	one, so we say one over six
T (26:20)	next: multiplication. First step is fraction times fraction. (.) first thing: do we need the same denominators? No, ya!? It's only by addition and subtraction, the same denominators. So what do we have to think?
S4	uh, we do, both
T	so you would multiply this here, the numerators, and then? Multiply the? Denominators.
S4	yes.
T	but before I would do that – that's correct – I would do something before!? This is the thing- ya: what would you do? Just say it!
S4	(?) you can
T	simplify. The three and the nine, right!? Even if it annoys you, please do it. Because later on, I do not want you to simplify at the end. If it is possible to simplify before you multiply: do it! (…) but later on, you will have numbers that are higher and, don't forget, you'll have algebraic fractions. (…) if you have this, it looks means, but here, you could simplify because if you, here, we have the distributive law, so if you

	would now, you would have to multiply the brackets out. But if you see that down here is the same as up there, then you can cross it out, and then you're already finished. So here, with numbers, it is easy (…) but [with] algebra, it's the same system only with brackets and with numbers and with letters.
S8	Like in the example, then you have no- like, is it like a times zero, and-
T	no: it's one.
S8	oh, it's one then?
T	ya, because it's: how often does b plus one fit in b plus one?
S8	one time
T	ya. And that's why it's one. So it would be, so you have a times one then, and one times a plus c (…) so this is the reason why you have to simplify before you multiply out. Because if you don't do that, you have such a fraction at the end because you have to start to multiply out everything. This is the reason why you already do it here.
S8	(so in the example with) three over five times four over nine, so you would cross the three out and put one in and cross the nine out and put three in, right!?
T	yes, because three fits one time into the three, and the nine fits, uh, the three fits three time into the nine.
S8	is it then in divided? The- ya, when you, where you put the-, where you turn the numbers around
T	ya, that's division (…) So: first: simplify, and then multiply! And numerator times numerator and denominator times denominator
(34:40)	ok, as a second we have: mixed number times mixed number or fraction, of course. What's the new thing we have to think here? When we have mixed numbers? What do we have to do before we start?
S7	convert them?
T	yes, first: convert!, then simplify, then multiply.
(26:30)	ok, last one: division! For example: four over fifteen divided by eight over three. L. already said it: what do we have to think about? The first thing division: oh! The first thing that has to shoot into your brain is what? Which information?
S1	that you have to swap the denominator with the numerator from the second-

T yes, the second one, never the first one! The second one! Reciprocal, it's called. So reciprocal. Second fraction: swap numerator and denominator! So what we do is this here, we swap, and what happens as well?

S7 uh, it turns into a times.

T yes. It turns into a times. So you actually multiply by its reciprocal

S2 do you always change the right one or the left?

T what?

S2 like the-

T second fraction. Never the first one. Always the second.

S2 and if you have four over fifteen divided by eight over three divided by six over eight!?

T then, you have to do it step by step. The first one stays, the second one, you turn around and multiply it out, the next one you turn around and then you multiply it out.
Ok, division sign turns into a multiplication sign . (…) mit dem umkehrwert mal nehmen. so you just multiply with the reciprocal. (?) multiply by.
Ok, and then!? What do we do then? Now we have, we multiplied by the reciprocal, that means: what do we have to do? What's now?

S6 shall I say the answer, or?

T no, what do we have to do now? So we have this here: this was the task, then we swapped the numerator and the denominator, and what's the next step?

S6 we have to have the same denominator?

T na, what is this? Which sign do we have here?

S6 times.

T times! So, now, at this step, we zoom over here: look! Do we need the same denominator?

S6 we have to simplify!

T we first have to simplify. And we don't need same denominators by multiplication, only by addition and subtraction.
So this is actually the only interesting thing by division: you swap, or multiply by the reciprocal and then, if you are at this step, you go back to multiplication and then you take the steps; first: simplify, then multiply, and then you're finished. So: this is the step for division, the rest is multiplication.

(44:00) (…) so, what you can do is: that, of course you have to think about that if you would show the three as a fraction, what is normally not necessary, as a- that helps you to visualize it, is three over one. That the three is divided into one part. (…) it seems to be a fraction but it isn't. this, you should never to in an answer because it doesn't look good. (…)where do I have to go with the three? Does it go up on top, does it go in the bottom? To help you: is, in the next step, that you say: I just convert the three into a – some sort of fraction. And it would be three over one. That you then know that you have to, that you actually multiply the denominator by three. So if you have a whole number, just convert it into this here.

S8 do we have to simplify before, like at the division sign or at the multiplication?

T multiplication. Never before.

S7 so, what should we do now with the (Scheinbruch)? Should the three go on top or at the bottom?

T so if you would display the three as a fraction, then it looks like three over one. And then: its reciprocal is one third

S7 oh, coz it's divided

T ya, here, it stays the same. So, the reciprocal is one-

S7 and if it would be times, it would be three over one!?

T three over one? What do you mean?

S7 if it would be four over fifteen times three, then we would-

T then, it would be- ya! Then you can say it's like that. Then, the numerator is tripled.

T (46:45) ok, last step is: mixed number divided by mixed number. What do we have to think about then? What's the first step?

S1 uh, convert!

T into what?

S1 into, like the-, (times, times)

T well, it's very similar. So: what do we do with the mixed number (?)

S1 into an improper fraction!

T ya, so this is then the first step. So first, before you even start, convert into improper fraction. Then, second step?

S1	then you have to change it to the times
T	reciprocal
S1	then you simplify and then you multiply.
T	and at the end, it doesn't matter if it's addition, subtraction, multiplication or division, what do we do at the end?
S2	we write it as a mixed number?
T	ya, so make sure there is a mixed number, not a improper fraction, it is simplified, that you have, that you displayed the fraction in its smallest, or easiest, simplest, simplest!, in its simplest form.

T	(?) ending decimals- what do you think?
S6	even numbers?
T	All even numbers? For example the six: what's with the six? It's even: would it be a ending decimal or not? One sixth, is that a ending, if you- uh, decimal, or a never-ending? What do you think?
S6	uh, it's, it's a never ending!?
T	it's a never ending. It was the rule, so if (?) always even, so: what was it? (?) What was the rule? How can we recognize (these) numbers?
S7	(all) numbers like, uh, (?) except three, except something that (?) all powers of two
T	this is (?) all powers of two fit into a multipl- uh a power of ten, so the two fits into the ten, the hundred and the thousand. The four fits into the hundred, so it is a factor of hundred. So, if you have multiples where the four fits-, all multiples of a three, for example, could never fit into a power of ten. Because of the three is never a factor of a power of ten. So the three doesn't fit into ten, neither in hundred, neither in thousand, neither in ten thousand. So that's why all multiples of three will never fit into these numbers as well. Get the logic. For example now, the six is even, so it is a multiple of two, but it is also a multiple of three. That's why the six can never fit into a power of ten, because it's a multiple of three.
S6	so it's never-ending!?
T	so it's never-ending. As soon as you have a multiple of, for example, four, there you could look at: so if you have a multiple of four, for example, eight, ya!? The eight is a even number first of all, is a multiple of four, and a multiple of two. the four and the eight, they fit into powers of ten. For example: the four

into hundred and thousand, and the eight fits into the thousand. So that's why, it will be a? (.) ending decimal.
If I have, for example, twenty, in the denominator, will this be ending or never-ending?

S1 ending.

T ending. Because the four and the five fit in and both fit into a hundred, a thousand or ten thousand.

S2 so if there's only one decimal, uhm one number that's (la-?) into the numbers, la-, la- so it's a never-ending

T ya, so as soon as you have a number where a factor never fits into a ten, a hundred, or a thousand. This number will never fit into a ten, a hundred, or a thousand. That doesn't work out, ya? So if, as soon as it is a multiple of three, for example, a multiple of seven, a multiple of six, this number cannot fit into a power of ten. That's the logic.
It was just because we had the decimals, uh sorry, the fractions, and we calculated and saw how many – which ones are ending and which ones are non-ending. Because we had two new words and the ones is rational numbers and irrational numbers.
Rational numbers can all be always displayed as a fraction. So even if I would have a whole number, like two, I can display it as a fraction: two over one. So, every number that I can show or, as a fraction, is a rational number. So if I would have a decimal, for example, and the decimal is ending, or, now it comes: recurring and never-ending, so recurring and never-ending is something like zero point three three three three three or zero point one six one six one six. And that is a never-ending. Then, the number is rational. As soon as I have a number that I cannot display as a fraction, I cannot show it as a fraction, these numbers are irrational. And these numbers, I can also recognize when I show it as a decimal, these decimals are? Never-ending, and non-recurring. For example: which one is the most famous irrational number? It is the one you should never ever forget.

S1 five!?

T five? A whole number: five! Displayed. Five over one: fraction.
(.) Pi. Ya!? (…) and this number is a never-ending, non-recurring decimal. You would never see a pattern that, uh, will be, that repeats. So that's why this is a number that is irrational. And that's why they gave it a symbol because this is actually the most accurate way to display the number. Because as soon as we cut off or we shorten the number, it is not accurate any more. It is not accurate because we round it then at the end.

(07:15) if we have this here: [writes: square root of five], if you would now, if I put it in the calculator, and say: extract, then, the answer will be two point blablablablabla.

S2 twenty-seven.

T	twenty-seven?
S2	no: two point twenty-three.
T	two three? Two times two. no, I would say it's two, then we have a four, three. Two point three, isn't it? Look into your calculator!
S2	I'm just guessing.
T	you're guessing!? But Look: two times two is four, already, and then you have to have a three because then you go up to – am I thinking right? Type in your calculator: extract the root of five.
S2	ya: two point three zero one.
S3	two point two three six seven nine
T	ah: two three six! Blablalabla. So: this is a number when you look at it, it is again a decimal that is never-ending and non-recurring. So that means: this number is a irrational number. So please, if you have that, leave it in this form: leave the five in the root, accept it like that and don't calculate it. Because this is the most accurate form to show the number. As soon as you, again, shorten the decimal, or the number, then it is not accurate anymore.
(08:45)	What is with this number? [writes square root of nine] Do you remember? We had that in year eight.
S6	it's three.
T	so: is the root of nine, the square root of nine, is that now a rational number or a irrational number?
S4	what?
S5	rational!?
T	why? (.) common sense: what kind of a number is this?
S5	a whole number!?
T	it's a whole number.
S4	it's a prime number, or?
T	it is a-? it's a prime number, yes. Ya, that's true. And? Can you display the three as a fraction?
S5	no!?
T	of course!
S5	oh, ya.

T	Scheinbruch (…) you could display it as a fraction. It looks like a fraction, but it's-
S5	Can't you display any number as a fraction? Except the – ah – except the (?)
T	(this is not a fraction) ya!? That's the problem. So, please remember, as soon as you have a square number in the square root, you are allowed, or you have to, extract the root, because you have a integer as an answer. If there is not a square number in the root, then you don't extract it because this is the accurate, most accurate, form to display the number.
(10:40)	What is, when I have something like this: [writes: the square of square root of five]
S1	ah: it's five
(.)	
T	it's five? The answer is five!? So it's irrational or rational?
S7	(it's rational!?)
T	why, why is it rational?
S7	because it ends.
T	how do you see that it ends?
S7	because the root is being destroyed by the two.
T	so you mean that the two attacks the two, or, on the root here, and then it disappears. But could you explain why it disappears!?
S7	because, it's like, well, for example: let's say that there is three instead of five, over there. And it's like: oh! Nine, nine, I meant nine: and then root of nine is three and three up to the power of two is nine. That is just for example.
S1	so it's five there!
(.)	
T	yea, but here, you have a square number in the root and you extracted the root first and then you squared it. How do you do it-
S1	it's five!
T	how five? How do you get the five?
S7	so: five, is, well, we can simplify as root of five multiplied, uh, times by root of five.
T	ok! So: that's the long form because it's squared. So that means you have a root of five and you times it by root of five. Ok, so what is root of five times root of five? You are allowed to multiply it.

S1 (square root of) five.

T no. what is five times five, guys?

S1 twenty-five.

S7 oh ya: root of twenty-five, ya!

T so, ya: you are allowed then to put the multiplication, this is new, then you have this here, and then you extract it, and then it's five.

S1 (so I was right!)

T ya, but you didn't know why, you guessed

S1 it wasn't guess

T but how did you know that it is five?

S1 I don't know if it's right, when I just think, but it's, first, you extract five, so it's this two point what?

T blablabla, ya!?

S1 and then you just, you just square it, square it by- and then it's five.

T ok, then it's- ya. If you do it that way, ya.

(…)

S1 but isn't it always the same? Isn't always- you always have this. (this nine, and then you have to extract it). And it wont be the nine (left, right?). and if you have a, what?, thirty in there, it will (?)

T again: if you have thirty in here? Then, yes. Because the clue is: the root, the number in the root, is actually squared, because if you write it down in the long form, you can put the multiplication fact under one root and as soon as you multiply a number by itself, then you will always have a square number at the end. And that's why it is rational. Because, as we know, as soon as you have a square number in the root, then you can extract it

S1 oh, so that's how you can find out that it's rational!?

T ya, so actually, it doesn't matter if you have a rational, uh, if you have a non-square number in the root. As soon as you square it, you have a square number.

(…)

T (17:00) this is the value, that they say that this is worth this.

(…)

(17:30)	[wrote: $\frac{\sqrt{18}}{\sqrt{2}}$] What do you think?
S7	nine root, uh, root of nine!?
T	ah, ya, because again, it's the same here: the multiplication, you can write under one root, the same, you can do with the division. So we could write it like this: eighteen over two, and when we halve it, then we have root of nine, and then it is three. So that means that this fraction is rational.
S8 (18:10)	I don't get it.
T	you don't get it!? Look: you have a square root of eighteen over square root of two.
S8	ya,
T	the rule is: when you have a division or a multiplication you're allowed to write this multiplication or division fact under one root. Like that.
S8	ya
T	so, and eighteen over two: do we like that in this form?
S8	no!?
T	is that the simplest form?
S8	no.
T	no!, ya!? So, if we would simplify this fraction then we have a nine. And what kind of a number is the nine?
S8	a multiple of three!?
T	yes, as well, but, what kind of a number is it as well?
S8	square!?
T	aha! And what do we know? As soon as we have a square number in the root, are we allowed- do we extract it then?
S8	(yes!?)
T	yeah! Because it's a three.
S8	can you give us like (?)
T	for irrational: we wrote it down

S8 is this irrational?

T this here! Is rational! Because it's a three at the end.

S3 so rational is if it ends, right!? And irrational (with no) end. Why do you then need like, end and never ending?

T no, as, irrational number is as soon as you have a never-ending, non-recurring decimal.

S3 I don't really get the difference

S8 here, it's (about) decimals!?

T ya! Is a decimal no number? (.) Look: a two is a number, ya, it's a whole number. If I would write it down as a decimal, then the two would look like this: [writes 2.0].

S8 aha

T so: this is ending (…) and I can display it as a fraction if I want to. If I would have something like [writes 126.25], ya, it is ending, so that's why it's rational. Because I can display it as a fraction, even as a mixed number-
So, if I have something like this: [writes $12\frac{1}{8}$], it's rational because this is already a fraction and if I want to, I could also convert this into a fraction as well. Now: these are all ending. Can you remember, now the tricky thing is this here, for example: [writes $\frac{1}{3}$]. We know this is a special one because it is? A? decimal!?

S3 never-ending?

T never-ending. But, what is it? It is? (.) you have three three three three. It is recurring. That means the three is never-ending because we have threes until we die.

S3 ya.

T that means: then we talked about that, because these are special fractions, because how can we display them? As a fraction! We are able to- we can show it as a fraction, we'll do another one: [writes $0.\overline{78}$] zero point seven eight. It's never-ending, coz we always have seven eight seven eight seven eight. Normally, we would always convert the decimal into a fraction with a power of ten in the denominator. That's what we learned. Now: in this case, it is not possible, because it is a never-ending, recurring decimal. But, we are able to display it as a fraction (?) special fractions, what do we have in the denominator?

S9 nine? Or like nine nine

T	ninety-nine. So in this case, we have, is it seventy-eight over ninety-nine. And that's why we can display it as a fraction, and that's why this number is rational. All these numbers, even if it is a integer, if it is a whole number, or it's a mixed number, what kind of numbers we have, here: these ones, or even the recurring, never-ending decimal, we can convert them into fractions. If we now extract, if we look at Pi, this is a number that's, where you will never have a pattern, you always have different digits. So that's why this fraction, uh, this number, you could never convert into a fraction. That doesn't work. So that's why this is irrational. Again, back to this here: the rule is: you have two numbers in a root that you divide, you know algebra, so when we have a division fact or a multiplication fact, we are allowed to write it in one root. Like that. Here, we simplified it and now we have here a square number and that's why we can extract it and this is a whole number and that's why it is, all is rational.
(…)	
S1 (26:15)	rational means that it's not ending!?
T	it is ending. It can be ending. It can be ending or, when it is never-ending, then it has to recur.
S1	rational means that it can end, so
S8	gosh! Now I don't know what-
T	for example: zero point three three three three three: never-ending. Is rational because it is a never-ending decimal, but it recurs, you always have the same pattern.
S1	ya, but it means that it is re, uhm, that it is never-ending. (you said) it can be never-ending.
T	it can be. You can also have a decimal that is ending, for example zero point-
S1	so this would also be a rational number!?
T	yes.
S2	and what is then irrational?
T	when it is never-ending and non-recurring.
S2	ah
S8	can we do this with a calculator?
T	it helps you? Will tell you if it's rational or irrational?

S2 yeah, but now, like in one, for example, we said that we should like, because you can't do it in a more accurate way, so it should be- just stay like this, so-

T you, just say, for example, pi over two, is it rational or not?

S8 no!?

S2 no

T no. then write down irrational

S8 but how do we know that it's not?

T well, pi: what kind of a number is pi? That's easy.

S8 pi: three four-

T no, what kind of- no, is it rational or irrational?

S8, S2 irrational

T irrational. So if you divide it by two, do you think that the decimal, that the never ending, non-recurring decimal will be suddenly never ending and recurring?

S2 also like if irrationals (?) a really long number but it's not recurring.

T ya!

S8 ya, but-

S2 recurring is if it's like, uhm, three three three three, or three one three one three one

S8 ok, if it's that, it's not rational

S2 if it's like- well, then it is. If it's like a pattern, then it's rational. But if it's not that, then it's irrational.

S8 so number one: it's not.

S2 it's irrational.

T (28:40) now, you could extract now, for example, this here. Use your calculator. And then you can look how your answer looks like

S2 (29:23)	(…) (?) ya, but I could, uhm, for example in bracket seventeen and then bracket and then the square - number
T	what happens with the seventeen when you square it? What kind of a number do you have?
S2	so seventeen times seventeen-
T	is? (.) blabliblu
S3	but it's (so it's) (rational)
T	no, the number is still in the root. But what kind of a number is it? When we times seventeen by itself?
S2/3/4	irration?
T	we don't know the answer. What kind of a number is it?
S2/3/4	(square number?)
T	aha, coz it's seventeen times seventeen, ya!? As soon as you multiply a number by itself, you have a square number. And when you have a square number in the root, can you extract it?
S2/3/4	no – ah yes.
T	no? yes! And what kind of a number will you have as the, as an answer?
S2/3/4	seventeen.
T	ya, so: irrational or rational?
S2/3/4	rational.

T (30:45)	pi is what kind of a number?
S1	irrational.
T	irrational. So, if we divide a irrational number by two, how does the answer look like?
S1	irrational!?
T	it stays irrational, ok!? So try with the calculator: press the button pi and divide it by two. what happens? How does your answer in the calculator look like?
S2	one point five seven

T so, is it a never-ending, non-recurring decimal?

S3 ya.

T ah. So: it is irrational. Now, if we have root of five, square root of five, if we extract this root, how does the answer look like?

S4 (?)

T so what kind of a number is it?

S4 irrational.

T it's irrational because? The answer looks like how? How does the answer look like? It's a decimal that is?

S5 never-ending

T never-ending and?

S4 uhm non-

T non-recurring.

T (32:03) now: next one: seventeen, root of seventeen, and this is squared.

S8 it's rational!?

T it's rational. Because: take your calculator, type in the seventeen, and extract the root. First, extract the root, what kind of a number is it? When you look at the answer?

S1 never-ending!

T it's never-ending and?

S2 no recurring

T non-recurring! So, now: square it!

S3 seventeen!

T seventeen. So this number is? A?

S2 rational

T (32:45)	now, we have a three in the square root, if we would extract it, how would the answer look like?
Stds	irrational
T	why?
S1	coz it's non-recurring.
T	and?
S1	and never-ending.
T	and never-ending.

T (33:20)	now this number [writes $\frac{\sqrt{12}}{\sqrt{3}}$]: is it rational or irrational?
Stds	rational
T	why?
S1	(?)
T	the answer is two. because, again, the division fact, you can write under one root, and if we would simplify the fraction in the root, we would receive a square number and the answer is then two, rational.
S2	oh, it's three, I thought it's thirteen
S6	I thought it should be a fraction!?
T	it is a fraction!
S6	yeah, but you divided.
T	what is this for a sign here? In algebra?
S6	a division one. But when you have a question, like three over ten, you also don't divide it.
T	no, because that's the simplest form, that's why you leave it like that. But this one: you can simplify, it's not the simplest form. What's with pi squared?
S3	irrational!
T	of course!

T (34:35) what's with this here? [writes 3-1 + 3-2]

Stds it's rational!

T why? What's the rule? Indice rules.

S1 yeah, because of the indice rules, there is one over three and one over nine, and then, it's a fraction, so

T yeah, that's the reason, rational. (…) rules of indices, we're gonna have it again. When you have a negative exponent, and you want to get rid off the negative exponent, you have to use the reciprocal. And as you can see, the three comes down into the denominator and is then positive. Because the one, we don't write it down, is now positive up there.
What's with seven minus a half, or up to the power of minus a half?

S2 rational (?).

T ya?

S3 minus a half of what?

T no: seven up to the power of minus a half.

S3 oh, up to the power!!??

Stds up to the power?

T when we have a seven up to the power of a half, how would you write it down?

S7 four point one, uh, eleven point five

T if I have a fraction in, as an exponent, how can I write it in another way?

S7 ah! It's forty-nine divided by two.

T no.

S4 is it? I'm not sure if it's correct, but don't you do it, is it then seven, and then there's like zero point five?

T uhuh

S7 ah, it's four point, four point five.!

T but if you have a fraction in a exponent, how can we write it in another form?

S3 minus (twenty-four) point five?

T	no? (.) Ok, other way what you can try out is: calculate with you calculator, instead of a half, zero point five. So. What does the calculator say when you say: seven up to the power of minus a half?
S5	it says six point five.
(…)	
T (28:33)	this calculator says it's root of seven over seven.
S5	(…) then it says zero point three seven seven (?)
T	(…) ya. And? What kind of a number is it?
S3	ending.
T	no.
S3	it's irrational
T	it's irrational. Why? How does your decimal look like?
T2	wait: it is square root of seven over seven, because if you- One over square root of seven is equal to square root of seven over seven
T	when you have this here: then you have square root of seven, or not?
T2	right, but it's to, to the negative, so
T	but it's minus
T2	so it's one over square root of seven.
T	ya, so that would be the next step
T2	and one over square root of seven is the same as square root of seven over seven
T	like that!?
T2	right.
T	and this is?
T2	that is the same as square root of seven over seven
T	but if you look at this: would this be now rational or irrational?
T2	that depends on how many digits the calculator is programmed for

T	how can I change this here? Ah, ok: then it's ending!
T2	no! it's not! Ending.
T	this- ya, ok. So that's why this is not ending
T2	of course it's not
T	ya, that's what I'm saying. Ok, it is of course never-ending and irrational. But it doesn't work with this calculator.
T2	well, that's because the calculator is programmed to only give eight digits or whatever
T	ya, no: the difference is here, you can type it in, and I can say if the exponent should be negative or not. This calculator doesn't work
S8	but you can do it with this one, can't you?
T (42:40)	this one: rational or irrational? [writes $\frac{22}{7}$]
S10	irrational?
T	what kind of a number is this one? It's a?
S10	a whole number
T	uh? No!
S10	no, it's a fraction!?
T	it's a fraction. And, fractions, what are they? Rational or irrational?
S10	rational!
S6	but this is one thing I don't understand. I did now twenty-two divided by seven and it was irrational.
T	huh?
S6	it was irrational! Shall I do it again? Three point one four two eight five seven one four three
T	ya, but now, common sense: how often does the seven fit into the twenty-two?
S6	three times!?
T	mhm, and what is then left over?

S6	one over seven!?
T	aha, and how does the decimal of one over seven look like? (.) look: common sense, what did we do with the worksheet that we had? One over seven – how does the decimal look like? It's a fraction. First of all: all fractions are rational! Basta. But if we extract, if we would now convert it into a decimal, how does the fraction look like? We had that at the beginning.
S6	zero point one four?
T	no, I don't want to have the answer. Is it never-ending and recurring, is it ending?
S6	no?
T	look on your paper! Take the paper out. What we had last time: what did you write down by one seventh? (.) no, your homework (…) Look at the list: one seventh: what did you write down in the table? (…) So now, look, one seventh, how does the decimal look like?
S6	one point one four two eight five seven
T	mhm and how many cycles?
S6	four?
T	mhm, so you always have four numbers and these four numbers repeat and repeat and repeat and repeat
S6	ah ja!
T	so: as soon as you see a fraction, it is in general rational
S1	now what does in general mean?
T	always!
S1	it sounded as if
T	no, that's the good thing about maths, you never have in- when it is the rule, then it is always the rule, you have no exceptions. (…) for maths: it is that way and it will always stay like that
S1	but with prime numbers: all even numbers are not prime numbers except-!
T	ya, that's the only one because that's convention.

T (46:03)	root of two plus one. Rational or irrational? So, if you would extract the root of two, what kind of a decimal would it be?
S1	rational!
S6	irrational
T	irrational.
S1	wait-
S2	it's also irrational!?
(…)	
S2	of course it's irrational.
T	ya. And here: the last one [writes $\sqrt{2.25}$]
Stds	rational irrational
T	two hundred twenty-five!?
S6	isn't it two point two five?
T	ya, it doesn't matter if you have a two point two five or a two hundred twenty-five (.) Extract the root of two point two five! What's you answer?
S1	one point five
S6	one point five!
T	because fifteen times fifteen is two hundred twenty-five.

S1	what is a factor?
S2	the opposite of a multiple.
T	yes, exactly
S2	it's what you can divide a number by. And a multiple is what you get as a result when you multiply
T	very good. A fraction is if it fits into a number.

T (00:50)	a sequence is given by: the term of n equals to n, no, nine n plus two. calculate the first four terms of the sequence. What are the first four terms?
S7	eleven, twenty, twenty-nine and thirty-eight.
(…)	
T	so this is the first four terms. And now we have: how does the difference between terms relate to the formula? What does that mean? That's the clue how to create the formula.
S8	because it always adds nine and like the formula, is -
S9	I only wrote add nine.
S1	ya, me too
T	ya, add nine. And that's why you multiply the nine with the n. (it's) ok coz this is the first part of the nth term.

T (02:28)	a sequence is given by the formula term of n equals to eleven n minus seven. What would you expect to be the difference between the terms in the sequence?
S9	uh, eleven
T	yes, because the eleven stands in front of the n, so that's why we will always add eleven. (…) calculate the tenth term of the sequence: what do we have to do?
S1	one hundred thirteen!?
T	one hundred and thirteen – why? What did you do?
S1	uh, you take the sequence and times it by a hundred!?
S2	(I go a) hundred
S3	ya, I don't understand
T	what would, how would, which sequence times hundred – what do you times hundred? Or times ten, actually.
S1	no: eleven times a hundred
S4	ten
T	no: ten, it's the tenth term.

S1	ah ja!
T	so: that is hundred ten. And what do we do now?
S1	minus seven
T	minus seven, ya!? And then we have hundred three.

T (04:50)	what is the difference between the terms for the sequence?
S7	minus four!?
T	yes because we can see that it decreases by four. How does this difference relate to the formula?
S7	minus four n
T	calculate the twentieth term of the sequence, what do we have to do?
S10	well, it's just, the twentieth; we do eighty-two minus in brackets four times twenty. And then-
T	then it's two. (…) because of eighty-two minus eighty.

T (07:20)	which sequence is decreasing? (…) why?
S10	because it's le- because you minus, and the, like the nth term is on the right
T	ya. So it's negative. You have a negative number in front of n.

T (09:20)	so: pattern number one looks like a cross with four tiles, and pattern number two, he attaches another four, pattern number three another four and pattern number four also another four. (…) what is the first picture and what is the differences to each pattern. And then you already have the sequence and you can already write down the numbers. (…) How many tiles will Owen need altogether to make pattern number six?
S11	(twenty)
T	mhm, because he always have four more. So it's an multiple of four.

T (14:10) It is possible – determine a formula for linear sequences. Sequences, where the difference between successive terms is always the same. This is linear. Otherwise it is not linear. The first difference is, for the number – (…)
So, we'll start. (…)
(15:30) so, the first thing is: that's the clue by linear differences is: first step is: you look at the sequence and you first find out the difference. So if we look at the sequence in number one, seven eleven fifteen nineteen, what is difference between these successive terms? What's the difference?

S11 four

T four. So, what is the first part of your formula?

S11 (four n)

T (16:15) perfect. (…) so, you add four. It's always the same difference. So, the first part of your formula is four n. you have a four in front of the n, so that's why on top, you write the multiples of four: four, eight, twelve. And then you look at the difference. So if you would have four, what do you have to do to get seven?

S11 add three?

T yes, so you would have to add three. And this is your second part.

S10 so four n plus three!?

T ya, and then you're finished.

T (17:45) ok, what is the formula? First of all, what do we look for?

S12 for where?

T here

S12 ok, uh, so (.)

T first, we find out the-?

S12 you- add four?

T we add four. So, what do we know now?

S12 that the-

T - first part of our formula is?

S12 is four n.

T four n. good. What do we do now?

S12	(.)
T	so we have a four now, so what you do is: you write the multiples of four up on top. Now, look at: so you start with the four, what do you have to do to get the six?
S12	uh, add two!?
T	add two. so that's the second part of your-
S12	ok, ya
T	and then you're finished.

S13 (19:00)	ya, you add two.
T	you add two. so what is the first part of the formula?
S13	uh, two n!?
T	two n. what do we do now?
S13	now, two times one
T	is two. and here would be four
S13	uh, plus nine.
T	plus nine. Ya!

S10 (19:48)	so, it's always plus seven
T	so what's the first part?
S10	the first, it's, uh, plus seven
T	seven what?
S10	n?
T	yes! Seven n. ok. And then?
S10	and then, plus- (.)
T	what do you write above now here?

S10 wait- (.) seven fourteen?

T seven f- yes! The multiples of seven up on top, here. Ya!? And now look at the difference: what do you have to do? When you have a seven, you start by seven, what do you have to do to get nine?

S10 divide?

T no. divide? You have seven and you want to get nine?

S10 plus three, plus three!

T you have seven sweets, you want to have nine, how many do you need?

S10 oh no! plus two, plus two! I'm sorry

T ah! Plus two. and then you're finished.

T (23:25) Now, we had sequences where we just have to find out the difference, that we knew the first part, how often we have to multiply the term and then we had to look how much, how many, we have to add or subtract.
This is easy because the difference is always the same.
Now, the next higher step, is (…) are quadratic sequences. They are a little bit more tricky and this is more (.) IGCSE level. (…)
So, in ten point four, we dealt with sequences where the differences between the terms was a constant value; of course, always the same difference.
On this section, we extend this idea to sequences where the difference are not constant.
(…)
They say, for example, calculate the first six terms of a sequence defined by the quadratic formula. The quadratic formula, that I haven't introduced yet, you will have this definitely this year, and it looks like: term of n, n squared plus n minus one. So, and now, they want you to find out the first six terms. How would you find out the first six terms when they would give you this formula? Does somebody have a idea?
(25:55) (…) In maths, it's always the same system. So, maybe think about how did you find out the first six terms by linear? And it will be probably the similar method to find it out by quadratic.

S8 you mean n two, uh, n square plus n minus one?

T yes, and they want you to find out, this is now a quadratic formula,

S9 but the squared has like (?) add two squared to (?) right?

S8 so, if you start with one, then it's like one squared is one, then plus one, then minus one, then you have one!

T ya! So the first number, the first term is one.
So, to find out the second term, what do we do then?

S12 we do two n square

T na, not n! the two, here, you substitute, that means you replace, you don't have the n anymore.

S12 ok, so, two square

T two squared

S12 plus n

T na, what is n now?

S12 uh, plus two.

T two.

S12 minus one,

T minus one, ok. What is four squared?

S12 uh, four!

T four. Plus two minus one is?

S12 uh, five!?

T five. Everybody get this? It's the same system. So you just have to substitute the values of the term, or the number of the term for n, and then you find out the value of the sequences. Ok, how do I find out term three?

S11 three square? Plus three minus one.

T good. Three squared is?

S11 nine?

T plus three minus one is?

S11 eleven!?

T good. (…) n squared plis n minus one. Fourth term

S10 ah, so it's four squared plus four minus one, so it's sixteen plus four, ya, minus one.

T and the answer is?

S10	nineteen.
T	so, if we look at this, the sequence, if we would write the numbers beside each other, the sequence would look like this: and if you would now look at the differences, what do you see? What is the first difference? Between one and four? (?) What's the difference between one and four, uh one and five?
S7	four.
T	what's the difference between five and-
S7	six!
T	aha.
S7	uh, eight.
T	so, if you look at the differences, what do you notice? (?) You always- ya, you add two more than before, ya. And, uhm, is this now successive? Is this always the same amount? Is this linear, then? No! ya? Because you can see- this is how you can recognize the quadratic ones, or, because we do not add the same amount, it always changes
S9 (30:03)	so linear is that you always add like (?) four, four, four
T	ya, so that's why it's linear. Because you always, you go straight, always straight away. And quadratic, is like that:
S8	then why are they called quadratic?
T	because this is the, as they say, this is the quadratic formula, and you have now, because we have, when we want to find out this formula, we have to, first of all, have two steps, what we have to find out the formula, if you have n is first squared, that's why it's quadratic.
S8	so, all the quadratic formulas are with squared, or not!? Or can there also be, can also be without square?
T	no, then it's not squared. If we would now look at the curve- Can you remember what we did in year eight with m x plus b? so, if we would look at this here: y equals m x plus be, ya!? It is, if we would display it as a curve, or as a line, it is a straight line. That's why it's called linear. If we use, this is later on, what we're gonna have as well, it's in this thick book, it's called the quadratic formula, where you have x is first squared, then you have a x plus b. so you have a x value that is squared, then the normal x in the way we have it up there and a constant. And if we would now find it out and display it as a curve, then, it would first of all, look like that. So it is a curve. So, what do you think, how does a cubic formula look like?

S1	uh, with, power of three?
T	ya. So, if the x value is cubed then it's cubic.
(…)	
T (37:55)	ok, so, if we now look at the differences, we see that we always add even numbers and we always add two more than before (…) comment on your result you obtained. what they say is: note that the difference between the first sequence are constant. (…) so if you look at, the first time, when you look at the differences, we already saw, ok, we always, we start with four and we always add two more. If we now look at the second step, because we have two n values, one n value squared and the other one is normal, we could look at the numbers we added, and now look at the differences between these numbers. And if we do this, then we find something out and it looks like this: what do you see?
S8	Isn't this linear, again!?
T	ya, they're constant. So, in the second step, it's, the differences are the same.
S1	I don't get it.
S8	(but do) you have to do it as well, then? Like-
T	ya, also in this case, in the quadratic-, we always do it two times. First of all, you find out the first time, the differences, and then you look at these differences and find out the differences of the first differences. And in the second step, it's normally, then they are constant.
S1	and how do we write it down? Like this?
T	ya, but we're gonna practise that.
S1	ya, but how should you write it down if they ask you
T	yes, we will, first we will go through this and then we'll see how we write it down. (…) So, by quadratic differences, you have to find out, the first time, the differences of the original sequence, and then you find out the differences of the first differences.
T (40:40)	(…) the first terms of the sequence defined by the quadratic formula. And this formula is called three n squared minus n minus two. so what they did, same procedure, that's always the same, if it is linear or quadratic, it doesn't matter. To find out the terms is that we substitute, we replace the numbers of the term for n and then we find it out. And in this case, the sequence would be zero, eight, twenty-two, forty-two and sixty-eight.

	(…) determine the first and second differences for the sequence. So that means you write down the original sequence you found out, find out the first differences, as you can see, the difference between zero and eight is eight, the difference between eight and twenty-two is fourteen, then the third difference is twenty, and the fourth difference is twenty-six; and if you now find out the second differences, then you see that you always add six. (…) so, what happens in the first difference? If you would try to explain it: what happens there? First of all, do you add the same amount?
S1	no!
T	no. so if you look at it, what happens?
S1	you always add six?
T	six more than before, ya
S1	ya
T	so, what can we say about the second differences?
S1	you always add six.
T	ya, so, it's, there, they are constant. You always add six. Ok, note: for a sequence defined by a quadratic formula, the second differences will be constant. And equal to twice the number of n squared. Get it? So what you can see is, in the first example, we had n squared plus one minus one and the difference was two. ya!? Now, if we look at the second example, we had the task three n squared minus n minus two and the second difference was six.
S2	(?)
S3	so it's the first number always?
T	ya, do you get it? So if we look at the third one, it is five n squared minus n plus seven
S3	so the first-
T	and the second difference is ten
S3	so the first n is- the first n is always one!? Like in one sequence, we have: the first n is always (one)
T	ya, depends what you have – what do you find- what they want to say is: if you look at the quadratic formula, you can already see how the difference is, how the second differences look like.
S3	what is the first (difference)?

T	the first difference is where you see that the amount, what you add, increases. And to find out by how many, the amount that I always add, increases, I find out the second difference.
S3	so, if n in, let's say the third one, the first n you should use to find out the first difference? For example, you (?) with one, so it's n square, so one squared is one and (five time ?) is five, minus one four and then plus seven, eleven. So you (?) and then the next time you do it with two. right?
T	where are you? (…) which example now, five n squared?
S3 (45:10)	ya. So, the first one, the first thing you have to do isn't it like the number one, so. one in every n. and then in the second one, you do two in every n!?
T	yes
S3	ah
T	it's the same system. So the way you had, you found out the first term by the linear sequence, the same system you use for the quadratic. It doesn't change.
S3 (45:28)	and in the quadratic thingy-
T	quadratic formula!?
S3	ya, the difference will always rise by the same amount (?) than in the second difference!?
T	that's now the thing what you now have to look, look at the formulas again. They say n squared plus n minus one, the difference is two. now, the second example what they show us is three n up to the power of two minus n minus two, the difference is, the second difference is six. Now we have five n squared, minus n plus seven, and the difference is ten. So, for example, if I would have four n squared, what would be the second difference?
S3	four n?
T	ya.
S3	like,(why four n?)
T	ya, that'y my question. I want to see if you get the system. Look:
S4	only if you have two every- (sequences?)
T	you have n squared, ya? This is new: this is the quadratic formula that you have a n squared. And we just want to find out: what is the second difference? So, because we know we have two differences.
S3	but they(?)

T	what is the second difference?
S5	two.
T	two. so, second difference is two. now they say, if I have a three n squared, plus n minus one, for example,
S5	six.
T	six! Why? How can you see this? So we know it because we found out that we calculated the sequence, but, for example, the next example was, what they have is five n squared plus n minus one, and they say the second difference is ten. How do they know that?
S14 (47:35)	ah, you have to times the n by two!?
S9	wait, what?
S14	well, no, I mean, uhm, in the first one, you have n and the squared on top, so you have to do one times two!? so you have two. and if you do it on the second
T	one squared, you mean!? Not times two.
S14	no-
T (48:00)	the small tiny two says that you
S14	well, if you say times two, because it will work for every one, (too). So: now, on the second one, for example, with, just replace, like, the three with the n, ah no: replace the n with the three, I mean, and (have that calculate ?) and if you do it with the five, replace n with a five, it will be, kind of, (two equals)
T	see it? So if we would have, for example, two n squared plus n minus one, what would be the second difference?
S14	four
T	four. Do you see it? L. got it. How does he know, without replacing for-
S9	I'm so confused-
T	look, you could find it out by finding out the first five terms. You could find out the second difference. But L. skipped this and said now, just by looking at the formula, the second difference is four.
S14	you just take the first number, and if there's like no number like in the first one, it's one. So it's one times two, always times two, ok!? (…) and on the second one, it's three times two, on the third one it's five times two, always times two, the first number.
S9	why times two?

S14 because of the nth term, isn't it?

S9 but I thought squared was – uhm –

S14 I know, but that's how you-

T that's just the-

S14 that's what you see

T that's what you see!

S9 oh!

T look, they could give you now this formula: and to find out the second difference, one possibility is that you first find out the first four terms. So, to find out the first four terms, you do it – what we did the last times over and over again, that we say, ok: term one is one: one squared plus one is two minus minus one is one. Ya? So, the second one would be two, so four plus two is six minus one is five. Nine plus three is twelve minus one is eleven.

S9 but why nine?

T nine? Because three squared is nine. (.) ya? It's the third term, we substitute a three here for n, and then we have to square it.

S9 and n is- number- like when you

T of the term, ya.

S9 ya.

T that's what we did, same system. Now, what we could do is, that we first find out the first difference, in this case, it would be four and it would be six. And then, we have to find out the second difference. And the second difference is two. so, we could find out the second difference each time by finding out the first terms of a sequence, the first step would be: find out the first difference and then find out the second difference. Or the really short way is: you look at the formula and you see some sort of pattern. And that's what L. already found out: he saw the pattern. Because he said: the second difference is two and he said, well, a one is in front of the n, and I times the one by two. and the second difference is two. So, here in this case is a three in front, and L. said: well, I just double the three and I know the second difference is six. So he just skips this step. He just looks at the first term, first part of the quadratic formula. This is important because when you do it the other way around, what we're gonna do, then you have to know the system.

S14 (52:45) but there are different sequences like, when you have a sequence, for example, with always times two, then like, four, eight, sixteen, thirty-two, how would you-

T yeah, that's a other one, that's a other sequence. We're now by quadratic sequences. (…)

S1 but now, also the second one isn't like linear, so what do we do then – third one?

T the second one!? That's what they say: by quadratic formulas, if we have a quadratic sequence, then the second difference is always linear.

S1 but this one isn't linear.

T which one is not linear?

S1 (?) have to be the same?

T no! look! It depends which number you have in front of the n squared.

S1 ya

T so of course, it changes then.

S1 ah ja, ok!

(…)

T this note, what they have here, for a sequence defined by a quadratic formula, the second differences will be constant and equal to twice of the number of n squared
(…)

(55:00) so, again, the second difference by a quadratic formula is always constant. And we could find out the second difference by looking at the quadratic formula, and the system is, that we look at the number in front of the n squared and multiply it by two. so, for example, three n squared, the second difference is six.
(…)
Determine or Generate a formula for the general term of the sequence. So, what they do- what you have to do- is this: so you have two, nine, twenty, thirty-five and fifty-four. So, the first thing what you have to do is, first: find out the first difference. So, the difference between two and nine is seven, then we have eleven, then we have fifteen and then we have nineteen.

S15 four!

T so, second step is: because it's a quadratic formula, we find out the second difference. And the second difference is?

S1	four
T	four. So now: this is the clue, what is the first part of our quadratic formula?
S15	(.)
T	so, look at this number: the second difference is four, now: think the way back. How does the n square term look like?
S2	it must be something like- uhm
S16	I don't understand the sequence because there is something added!? I don't understand the sequence.
T	why? It is a quadratic sequence, that's why it's now different, because we don't add the same amount, that's the clue.
S16	achso: ok.
T	that's why we have now two steps to find out the constant. The constant is now four. How does the first term look like? The n square term look like?
S3	why should (?) can't you just say: because the term- like the- (the thing they look for), can't it be like two n square plus four, or something?
T	two n- so, we're only looking at the first part, so: first part looks like two n squared
Stds	(??)
T	you say two n squared?
S3	it's written here.
T	yes! But that's what you have to f- but why?
S3	I thought, isn't that the question? Like to finish it- I don't get it!
T	no! this is an example (?) solution (…) Again, try to think: you have to now start to understand, or to think. So, the first step was that they said that a quadratic formula is that the second difference is always a- constant. Here's the proof again. At the first way, the one way, they said: look, when I have a one n squared, my second difference is? What is the second difference?
S4	two?
T	two. so, here, the constant would be two. because they said: the number in front will be timesed by two. so, if I have three n squared, I can see, the second difference will be?

S5	six. I have a question-
T	wait a minute, get this: remember, that was this way. Now, the other way, think the other way around, now, we don't know this, we know this. And now, they say: the second difference is four, how does n squared look like? Which number?
Stds	two, two, two
T	two! so, the first part of the sequence would be, when you would generate the formula, two n squared. That's the first part. We have three parts. We have the n square, the n, and the constant. But this is, first of all, the first part. So, either they give you this: they ask you the second- but that's too easy, in this case, they give you a sequence, you have to find out the second difference and then you have to think the other way around, back, so in this case.
S3 (1:00:20)	so, if the second thing would be eight, then it would be four n squared?
T	yes. So this is what they are trying to show you in this case. So, now, we want to find out the n. because the quadratic formula looks like this: n squared, then plus n, so. Now, we have to find out this part
S14	(…) can it be multiple thing, actually? Like, could it like, now, for two n squared, for example, plus n minus one or minus n plus three!? Could be both, couldn't it?
T	ya-
S14	ya! So which one do you have to always write down? It doesn't matter, or?
T	again:
S3	(?) but they make a new sequence
S14	so, two n squared –
T	so, we have now two n squared
S14	that is eight. Like for the first term, (it is) two. and then, it's, ya, eight, ya, and then you just do plus n minus one. Isn't it? And then you have your (answer). And you could also do minus n plus three, couldn't you?
T (1:01:37)	couldn't you!? Now this is the question: both ways, are they correct? Or is there a system we have to keep in mind? (…) now, this is the first part of the sequence: two n squared, because of the constant, and then we have the second part. And now, they say that you, and this is what we found out, so the first part. Now: to determine the rest of the formula, we have to subtract two n squared
S14	why?

T	from each term of the sequence, as shown below.
S14	so, you mean the, not the term (position) but the term itself!?
T	ya. So, we have the original sequence, ya, that's the original sequence we started with. Now, we found out the first part of the t- of the formula is two n squared. So that's why we now write down the sequence of two n squared. So it would be-? One squared times two is? Two.
Lu (1:03:05)	and then times two – four.
S14	and then square – wait! So, it's two times n squared, so it's eight
T	yes.
S16	no, four!
T	look: always the exponent first! You substitute a two. two times two is?
S14	wait: but, for two, like, first number, then you just do: two times two squared !?
T	no.
S14	why not?
T	the exponent! Bedmas! The exponents before multiplication. So always square first before you multiply
S14	so, it's four then, isn't it? Why did you write a two there?
T	because one squared is one, times two is two. it's the term one!
S14 (1:04:00)	oh ja, the term!
T	term two, term three, it's always the same system. Now, we have the third term. so, first square nine, times two is eighteen, then we have four, sixteen times two is thirty-two, then we have five, is twenty-five times two is fifty. so, we draw a line and now, we look at the differences, we always constantly look at the differences. And in this case, the differences is zero, ya? Between the two, uh, terms; the difference between the original term and two n squared is one, then two, three, and five. See it?
S3	isn't it four with the last one? Isn't it four?
T	oh, ya.
S16	so it's two n plus one, ya: two n squared plus one
T (1:05:05)	so, but you have to think the other way around. What do we want to have?

S14 why did you write a ough and a one and a two, and a three, and afour?

T because the difference between two and two is zero, ya? And the difference between nine and eight is one.
(…) L. thought already correct because he said, if I substitute here one, then I have (…) now, they say: the new sequence has a constant difference of one and begins with zero. So far, this sequence, the formula, is n minus one. Why n minus one?

S14 n minus one?

T ya. Coz this is now the next sequence you look at:

S14 because the difference of there is one!

T ok, and why now minus one, and why not plus one?

S14 because-

T now, think this away, and this is your new sequence, ya? Zero, one, two, three. What is the formula for this sequence? Because as you can see, the difference is always the same, so now, it's linear.

S14 uh: it's one!

T so, create now the difference, uh the sequence, uh the formula, for this sequence!

S14 oh my god!

T look at it: so, we always have the difference one, and now: say the nth term! so, again: now, back to linear sequences. So, the difference is one, so the first part of the nth term is? Huh n. so, which number? One, two, three, four, five-

S14 one.

T one n. so now we write down the multiples of n up on top. What do we have to do to get the zero?

S9 minus one?

S14 minus one. Ya.

T and at last, this is the second part that goes up here. So it's n minus one. And this is the formula, we generate

S16 but how do we get the minus?

S14 because-

T it's the same system what we did with linear

S9 (1:07:48) one and minus one is zero

T here, it's the multiples of one and then-

S16 ah, ok.

S14 so, always take the-